## Math 165 Exam Ia Lowman Spring 2010

In all problems you must show written work that leads to your answer. The grader should be able to tell exactly what you were thinking when you were solving the problem. Put a box around all final answers. If the grader has to guess which part of what you wrote is your answer then he/she should mark it wrong.

1. Use implicit differentiation to find the slope of the tangent to the graph of $\mathbf{8 x} \boldsymbol{y}^{\mathbf{2}}+\boldsymbol{3 y}^{\mathbf{3}}=\mathbf{2 4}$ at the point $(\mathbf{0}, 2)$.
2. Find the following limit: $\lim _{x \rightarrow \infty} \frac{44+33 x+100 x^{4}}{2 x^{4}+2 x+3}$
3. Use the chain rule to find $\frac{d y}{d x}$ if $\boldsymbol{y}=\boldsymbol{u}^{3}-\mathbf{3} \boldsymbol{u}^{2}+1$ and $\boldsymbol{u}=\boldsymbol{x}^{2}+\mathbf{2}$. Give your answer as a function of $\boldsymbol{x}$ only (i.e. no $\boldsymbol{u}$ variables in your answer). To receive credit you must use the chain rule and not some other method that works.
4. Given $f(x)=\sqrt{\left(x^{2}+1\right)}+e^{2 x+1}+\boldsymbol{\operatorname { l n }}(3 x+1)+x$ find the derivative of $f(x)$. For credit show intermediate steps that lead to your answer. i.e. show intermediate steps that still have derivatives that need to be done.
5. Given the cost to produce one unit is $\$ \mathbf{2 . 0 0}$ and the demand relation is given by $\boldsymbol{p}=\mathbf{1 0 2} \mathbf{- 2 q}$. Find the following and Box Your Answers:

- profit function $\boldsymbol{P}(\boldsymbol{q})$
- find the production level that maximizes the profit by setting the slope of the profit function to zero and solving for $\boldsymbol{q}_{\boldsymbol{m a x}}$.
- use the second derivative test to show that the critical number you found actually gives a maximum profit.

6. For $f(x)=x^{3}$ Use $f^{\prime \prime}$ to find the locations of any potential inflection points. Use $f^{\prime \prime}(x)$ to show that the points you found actually are or are not inflection points. You do not need to actually find the point, just its $\boldsymbol{x}$ coordinate and what kind of point it is.
7. $f(x)$ is a function with first derivative $f^{\prime}(x)=(x-1)(x-2)(x-3)(x-4) / x \cdot f(x)$ obviously has a critical number at $\boldsymbol{x}=\mathbf{2}$. Use the first derivative test to determine what kind of critical point is at $\boldsymbol{x}=\mathbf{2}$.
8. The function in the previous problem has a critical number $\boldsymbol{x}_{\boldsymbol{c}}=\mathbf{2}$ and a second derivative $f^{\prime \prime}(x)=\frac{3 x^{4}-20 x^{3}+35 x^{2}-24}{x^{2}}$. Use the second derivative test to determine what kind of critical point is at $x_{\boldsymbol{c}}=\mathbf{2}$.
9. For $f(x)=-\boldsymbol{x}^{\mathbf{3}}+\mathbf{6} \boldsymbol{x}^{\mathbf{2}}-\mathbf{1 0}$, use $\boldsymbol{f}^{\prime \prime}$ determine where the graph of $\boldsymbol{f}(\boldsymbol{x})$ is concave up and where it is concave down.
10. For some $\boldsymbol{f}(\boldsymbol{x}), f^{\prime}(\boldsymbol{x})=\frac{(2-x)}{(\boldsymbol{x - 4 )}}$ use $\boldsymbol{f}^{\prime}$ determine where the graph of $\boldsymbol{f}(\boldsymbol{x})$ is increasing, and where it is decreasing.
