Note Title 4/7/2010

Math 165, Sp'10, Lowman Week 11, Friday

- Anti-Derivative, Indefinite Integral cont.

$$\left.\begin{aligned} \frac{dF(x)}{dx} &= f(x) \\ \int f(x)\,dx &= F(x)+c \end{aligned}\right\} \text{Same}$$

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for each derivative rule can find the corresponding Integral Rule.

This was covered in previous lecture

$$\frac{d}{dx} x^n = n \cdot x^{n-1}$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

Simple Power Rules

Use chain rule to get general version.

$$\frac{d}{dx} u(x)^n = \frac{d}{du} u^n \cdot \frac{du(x)}{dx}$$

Chain Rule

$$= n \cdot u(x)^{n-1} \cdot u'(x)$$

, gives

General Power Rules

$$\frac{d}{dx} u(x)^n = n \cdot u(x)^{n-1} \cdot u'(x)$$

$$\int u'(x) \cdot u(x)^n \, dx = \frac{u(x)^{n+1}}{n+1} + C$$

<u>Note</u>: In the special case where $u(x) = x$, $u'(x) = \frac{d}{dx}x = 1$ and General Rules reduce to simple rules.

Observation: you can convert a simple integral rule to a general rule by replacing x with $u(x)$ and dx with $u'(x)dx$.

Example

Simple Power Rule:

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

$$\int u(x)^n \, u'(x)dx = \frac{u(x)^{n+1}}{n+1} + c$$

Summary of General Rules

$$\frac{d}{dx} u(x)^n = n \cdot u(x)^{n-1} \cdot u'(x)$$

Power Rules

$$\int u'(x) \cdot u(x)^n \, dx = \frac{u(x)^{n+1}}{n+1} + C$$

$$\frac{d}{dx} \ln u(x) = \frac{u'(x)}{u(x)}$$

Log Rules

$$\int \frac{u'(x)}{u(x)} \, dx = \ln |u(x)| + C$$

$$\frac{d}{dx} e^{u(x)} = e^{u(x)} \cdot u'(x)$$

Exponential Rules

$$\int u'(x) \cdot e^{u(x)} \, dx = e^{u(x)} + C$$

<u>Example</u> (that does not work!)

$$\int \frac{2x}{x+99}\,dx$$

Not ok.

Check if fits the Log Rule.

$$\int \frac{u'(x)}{u(x)}\,dx = \ln|u(x)| + c$$

need the numerator to be the derivative of the denominator.

<u>check</u> $\frac{d}{dx}(x+99) = 1$

This integral does not fit any of the above three rules. (Skip for Now.)

Example:

Perfect fit.

$$I = \int \frac{2x}{x^2+6}\,dx$$

× Not Power Rule

× Not Exponential Rule

✓ Possible fit for Log Rule

must check to be sure.

$$\frac{d}{dx}(x^2+6) = 2x$$

$$\int \frac{u'(x)}{u(x)}\,dx = \ln|u(x)| + C$$

$$I = \ln|x^2+6| + C$$

$$= \ln(x^2+6) + C$$

Note that x^2+6 is always positive so the absolute value can be dropped.

More Rules:

$$\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\frac{d}{dx} K \cdot u(x) = K \cdot \frac{d}{dx} u(x)$$

$$\int K \cdot u(x)\,dx = K \int u(x)\,dx$$

Constants pass through the Integral Sign

$$\Rightarrow \int u(x)\,dx = \frac{1}{K} \cdot \int K\, u(x)\,dx$$

"multiply by one trick"

Example:

$$I = \int x\,(2x^2+1)^3\,dx$$ try Power-Rule

$$\int u'(x) \cdot u(x)^n \, dx = \frac{u(x)^{n+1}}{n+1} + c$$

Check $\frac{d}{dx}(2x^2+1) = 4x$ need a factor of 4 to fit the rule.

mult by 1

$$= \underbrace{\frac{1}{4}\cdot}_{} \int 4x\,(2x^2+1)^3\,dx = \frac{1}{4}\,\frac{(2x^2+1)^4}{4} + C = \frac{(2x^2+1)^4}{16} + c$$

Example: need a factor of 4.

$$I = \int \frac{x}{2x^2+99}\,dx$$ check if fits Log Rule

$$\int \frac{u'(x)}{u(x)}\,dx = \ln|u(x)| + c$$

check $\frac{d}{dx}(2x^2+99) = 4x$ always positive

$$I = \frac{1}{4}\int \frac{4x}{2x^2+99}\,dx = \frac{1}{4}\ln|2x^2+99| + C$$

$$= \frac{1}{4}\ln(2x^2+99) + c$$