Math 165, Sp'09, Lowman Week 12 Monday

Note Title 4/7/2010

In section 5.1 The Indefinite Integral was explained as an Anti-Derivative.
Simple and General Integral rules were determined from the corresponding derivative rules.

Now in section 5.2 these Anti-Derivative rules will be used to determine

the integrals in the worked examples.
Note, in section 5.2 of the textbook
the authors work every integral
by using the <u>method of substitution</u>.
We will determine the same
integrals by usin the anti-derivative
rules from 5.1.

Here is a summary of the rules.

General Power Rules

$$\frac{d}{dx} u(x)^n = n \cdot u(x)^{n-1} \cdot u'(x)$$

$$\int u'(x) \cdot u(x)^n \, dx = \frac{u(x)^{n+1}}{n+1} + c$$

General Exponential Rules:

$$\frac{d}{dx} e^{u(x)} = e^{u(x)} \cdot u'(x)$$

$$\int u'(x) \cdot e^{u(x)} \, dx = e^{u(x)} + c$$

General Log Rules

$$\frac{d}{dx} \ln u(x) = \frac{u'(x)}{u(x)}$$

$$\int \frac{u'(x)}{u(x)} \, dx = \ln |u(x)| + c$$

$$\int k \cdot f(x)\,dx = k \int f(x)\,dx$$

gives the multiply by one trick

$$\int f(x)\,dx = \frac{1}{k} \cdot \int k \, f(x)\,dx$$

multiply one.

Re-Worked Examples from Sec 5.2
(do not use the substitution method)

Example 5.2.1

$$I_1 = \int \sqrt{2x+7}\, dx$$ Use the general power rule

$$= \int (2x+7)^{1/2}\, dx$$

need a 2

need $\int \underline{u'(x)} \cdot u(x)^n\, dx$

need.

check if fits Rule: $\frac{d}{dx}(2x+7) = 2$

use the multiply-by-one trick.

$$I_1 = \frac{1}{2}\boxed{\int 2(2x+7)^{1/2}\,dx}$$

$$= \frac{1}{2}\,\frac{(2x+7)^{1/2+1}}{\left(\frac{1}{2}+1\right)} = \frac{1}{2}\,\frac{(2x+7)^{3/2}}{3/2} + c$$

$$= \frac{(2x+7)^{3/2}}{3} + c$$

In reality the integral would be done like:

$$I_1 = \frac{1}{2}\int 2\sqrt{2x+7}\,dx = \frac{1}{2}\,\frac{(2x+7)^{3/2}}{3/2} + c$$

Example 5.2.2

$$I_2 = \int 8x \cdot (4x^2-3)^5\,dx$$ check if Power Rule works.

$$\int u'(x) \cdot u^n(x)\,dx = \frac{u^{n+1}(x)}{n+1} + c$$

$$\frac{d}{dx}(4x^2-3) = 8x \Rightarrow \text{perfect fit.}$$

$$I_2 = \frac{(4x^2-3)^6}{6} + c$$

Example 5.2.3

$$I_3 = \int x^3 \cdot e^{x^4+2}\,dx$$

need 4

Check if fits the exponential rule.

$$\int u'(x) \cdot e^{u(x)}\,dx = e^{u(x)} + c$$

$$\frac{d}{dx}\left(x^4+2\right) = 4x^3$$

$$= \frac{1}{4}\int 4x^3 \cdot e^{x^4+2}\,dx = \boxed{\frac{1}{4}e^{x^4+2} + C}$$

multiplying by one

Example 5.2.4

$$I_4 = \int \frac{x^2}{x-1}\, dx$$

try the log rule. does not work.

$$\int \frac{u'(x)}{u(x)}\, dx = \ln|u(x)| + c$$

Need the numerator to be the derivative of the denominator.

check: $\frac{d}{dx}(x-1) = 1$ found 1 but need x.

the multiply by one trick does not work here. It only works with constants.

This is an example where the <u>substitution method</u> might be useful. However, there are two different "tricks" that can be used

<u>Add zero trick:</u>

$$I_4 = \int \frac{x}{x-1}\,dx = \int \frac{x\overbrace{-1+1}^{+0}}{x-1}\,dx = \int \cancelto{1}{\frac{x-1}{x-1}} + \frac{1}{x-1}\,dx$$

$$= \int 1\,dx + \int \frac{1}{x-1}\,dx$$

Note: $\int dx = x + c$, Here is one way to see this: $\int dx = \int 1 \cdot dx = \int x^0 \cdot dx = \frac{x^{0+1}}{0+1} = x + c$

power rule.

$$I_4 = \int dx + \int \frac{1}{x-1}\, dx$$

$$= x + \int \frac{1}{x-1}\, dx$$

use the log rule

$$\int \frac{u'(x)}{u(x)}\, dx \qquad \frac{d}{dx}(x-1) = 1$$

$$= \boxed{x + \ln|x-1| + c}$$

Using the "Long Division Trick".

$$I_4 = \int \frac{x}{x-1}\,dx$$ - does not fit the log rule.

- use long division

$$\frac{x}{x-1} = \begin{array}{r} 1 \leftarrow \text{Quotient} \\ x-1\overline{)\,x} \\ -(x-1) \\ \hline +1 \leftarrow \text{Remainder} \end{array} = Q + \frac{R}{D} = 1 + \frac{1}{x-1}$$

Divisor

$$I_4 = \int\left[1 + \frac{1}{x-1}\right]dx = \int 1\,dx + \int \frac{1}{x-1}\,dx$$

$$= \boxed{x + \ln|x-1| + C}$$