Math 165, Sp'10 Lowman, Week 12 Wednesday

Note Title 4/7/2010

5.3 Section 5.2 Examples Continued

<u>without using substitution</u>

$$\frac{d}{dx} u(x)^n = n \cdot u(x)^{n-1} \cdot u'(x)$$

$$\int u'(x) \cdot u(x)^n \, dx = \frac{u(x)^{n+1}}{n+1} + C$$

Power Rule

Exponential Rule

$$\frac{d}{dx} e^{u(x)} = \frac{d}{du} e^{u} \cdot \frac{du}{dx}$$ Chain Rule

$$= e^{u(x)} \cdot u'(x)$$

$$\int u'(x) \cdot e^{u(x)} \, dx = e^{u(x)} + C$$

Log Rule

Chain Rule

$$\frac{d}{dx} \ln u(x) = \frac{d}{du} \ln u \cdot \frac{du}{dx}$$

$$= \frac{1}{u(x)} \cdot u'(x) = \frac{u'(x)}{u(x)}$$

$$\int \frac{u'(x)}{u(x)}\, dx = \ln |u(x)| + C$$

$$\int k \cdot f(x)\,dx = k \int f(x)\,dx$$

constant k (No variables)

$$\int f(x)\,dx = \frac{1}{k} \int k\, f(x)\,dx$$

multiply by 1 trick.

Example 5.2.5

must always write dx

$$I_5 = \int \frac{3x+6}{\sqrt{2x^2+8x+3}}\,dx$$

Try:
Power Rule
~~Log Rule~~
~~Exponential Rule~~

why! Not Log Rule

$$= \int \frac{u'(x)}{u(x)}\,dx = \ln|u(x)| + C$$

check log Rule

$$\frac{d}{dx}\sqrt{2x^2+8x+3} = \boxed{\frac{d}{dx}(2x^2+8x+3)^{1/2}}$$

Does not fit the log rule or exponential Rule

Try Power Rule:

$$I_5 = \int (3x+6)\,(2x^2+9x+3)^{-\frac{1}{2}}\,dx \qquad -\tfrac{1}{2} = n$$

Power?

$$\int u'(x)\cdot\big(u(x)\big)^{n}\,dx = \frac{u(x)^{n+1}}{n+1} + C$$

$$I_5 = \frac{3}{4}\int 4(x+2)\,(2x^2+9x+3)^{-\frac{1}{2}}\,dx$$

u'?

check

$$\frac{d}{dx}(2x^2+9x+3) = 4x+9 = 4(x+3)$$

Need this here

1st factor out 3

2nd multiply by $\frac{1}{4}\int 4 \cdots dx$

$$= \frac{3}{4} \cdot \int \underbrace{4(x+2)}_{u'} \cdot \left(2x^2+8x+3\right)^{-1/2} dx$$

$$= \frac{3}{4} \cdot \frac{\left(2x^2+8x+3\right)^{\left(-1/2+1\right)}}{\left(-\frac{1}{2}+1\right)} + C$$

$$-\frac{1}{2} + \frac{2}{2} = \frac{1}{2}$$

$$= \frac{3}{2\cancel{4}} \cdot \frac{\left(2x^2+8x+3\right)^{1/2}}{\left(\frac{1}{2}\right)} = \frac{3}{2} \cdot \left(2x^2+8x+3\right)^{1/2} + C$$

$$= \frac{3}{2} \sqrt{2x^2+8x+3} + C$$

Example 5.2.6 Try??

$$I_6 = \int \frac{(\ln x)^2}{x}\, dx$$

- Power Rule
- ~~Log Rule~~
- ~~Exponential Rule~~

~~Log Rule~~ $\int \cancel{\frac{u'(x)}{u(x)}}\, dx = \cancel{\ln|u(x)|} + C$

~~Exponential Rule:~~ $\int \cancel{u'(x) \cdot e^{u(x)}\, dx = e^{u(x)}} + C$

$$I_6 = \int \frac{(\ln x)^2}{x}\,dx = \int \frac{1}{x}(\ln x)^2\,dx$$

Check if power Rule $\int u'_{(x)} \cdot u_{(x)}^n\,dx$

$$\frac{d}{dx}\ln x = \frac{1}{x} = u'_{(x)}$$

$$I_6 = \frac{(\ln x)^3}{3} + c$$

end 12-1:00

Power Rule:

$$\frac{d}{dx} x^n = n \cdot x^{n-1}$$

Chain Rule

$$\frac{d}{dx} u(x)^n = \frac{d u^n}{d u} \cdot \frac{du}{dx}$$

$$= n\, u^{n-1} \cdot u'(x)$$

$$= n\, u(x)^{n-1} \cdot u'(x)$$

$$\frac{d}{dx} u_{(x)}^{n} = n \cdot u_{(x)}^{n-1} \cdot u'_{(x)} \quad \text{so}$$

$$\int u'_{(x)} \cdot u_{(x)}^{n} \, dx = \boxed{\frac{u_{(x)}^{n+1}}{n+1} + C}$$

derivative of this
is equal to
this

Question: why is $u'(x)$ not in answer?

$$2^3 = 2 \cdot 2 \cdot 2 = 8$$

There is No 3 in answer

$$\sqrt[3]{8} = 2$$

Similar to this

$$\int \boxed{u'(x) \cdot \; u(x)^n} \, dx = \boxed{\frac{u^{n+1}}{n+1} + C}$$

take derivative of all and get

$$\frac{d}{dx}\left[\frac{1}{(n+1)}u(x)^{n+1}\right] = \frac{1}{(n+1)} \cdot \frac{d}{dx}\left[u(x)^{n+1}\right]$$

no $u'(x)$

$$= \frac{1}{\cancel{(n+1)}} \cdot \cancel{(n+1)} \cdot u(x)^n \cdot \frac{du}{dx}$$

Yes $u'(x)$

$$= u(x)^n \cdot u'(x)$$