

5.3 Section 5.2 Examples Continued
without using substitution

$$\frac{d}{dx} u(x)^n = n \cdot u(x)^{n-1} \cdot u'(x)$$

$$\int u'(x) \cdot u(x)^n dx = \frac{u(x)^{n+1}}{n+1} + C$$

Power Rule

Exponential Rule

$$\frac{d}{dx} e^{u(x)} = \frac{d}{du} e^u \cdot \frac{du}{dx} \quad \text{Chain Rule}$$

$$= e^{u(x)} \cdot u'(x)$$

$$\int \underline{u'(x)} \cdot e^{u(x)} dx = e^{u(x)} + C$$

Log Rule

$$\frac{d}{dx} \ln u(x) = \frac{d}{du} \ln u$$

Chain Rule

$$= \frac{d}{du} \ln u$$

$$\cdot \frac{du}{dx}$$

$$= \frac{1}{u(x)} \cdot u'(x)$$

$$= \frac{u'(x)}{u(x)}$$

$$\int \frac{u'(x)}{u(x)} dx = \ln |u(x)| + C$$

$$\int \underline{k} \cdot f(x) dx = \underline{k} \int f(x) dx$$

constant k (No variables)

$$\int f(x) dx = \left(\frac{1}{k} \right) \cdot k \int f(x) dx$$

multiply by 1 trick.

Example 5.2.5

must always write dx

$$I_5 = \int (3x+6) \sqrt{2x^2+9x+3} dx$$

Try:
Power Rule
~~Log Rule~~
~~Exponential Rule~~

why! Not Log Rule

$$\int \frac{u(x) dx}{v(x)} = \ln |u(x)| + C$$

check log rule $\frac{d}{dx} \sqrt{2x^2+9x+3} = \frac{d}{dx} (2x^2+9x+3)^{1/2}$

Does not fit the log rule OR exponential Rule

Tag Power Rule: $\downarrow \frac{1}{2} \leftarrow n$

$$I_5 = \int \frac{(3x+6)}{\quad} (2x^2+9x+3) dx$$

Power? $\int \underline{u'(x)} \cdot (\underline{v(x)}) dx = \frac{v(x)^{n+1}}{n+1} + C$

$n = -\frac{1}{2}$

$$I_5 = 3 \int \frac{4(x+2)}{\quad} (2x^2+9x+3) dx$$

check $\frac{d}{dx} (2x^2+9x+3) = 4x+9 = 4(x+2)$

1st factor out 3
2nd multiply by $\frac{1}{4} \int 4 \dots dx$

Need this here

$$= \frac{3}{4} \cdot \int 4(x+2) \cdot (2x^2+9x+3)^{-1/2} dx$$

$$= \frac{3}{4} \cdot (2x^2+9x+3)^{-1/2+1} + C$$

$-\frac{1}{2} + \frac{2}{2} = \frac{1}{2}$

$$= \frac{3}{4} (2x^2+9x+3)^{1/2} = \frac{3}{2} (2x^2+9x+3)^{1/2} + C$$

$$= \frac{3}{2} \sqrt{2x^2+9x+3} + C$$

Example 5.2.4

Try??

$$I_6 = \int \frac{(\ln x)^2}{x} dx$$

Power Rule

• ~~Log Rule~~

• ~~Exponential Rule~~

~~Log Rule~~

$$\int \frac{\cancel{dx}}{\cancel{dx}} dx = \cancel{\ln|x|} + C$$

~~Exponential Rule:~~

$$\int \cancel{\ln} \cdot \cancel{e} dx = \cancel{e^{\cancel{\ln}}} + C$$

$$I_6 = \int \frac{(\ln x)^2}{x} dx = \int \frac{1}{x} (\ln x)^2 dx$$

Check if power rule $\int u' \cdot u^n dx$

$$\frac{d}{dx} \ln x = \frac{1}{x} = u' \quad \checkmark$$

$$I_6 = \frac{(\ln x)^3}{3} + C$$

end 12-1:00 \downarrow

Power Rule:

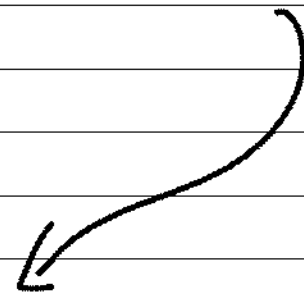
$$\frac{d}{dx} x^n = n \cdot x^{n-1}$$

$$\frac{d}{dx} u(x)^n = \frac{d u^n}{d u} \cdot \frac{d u}{d x}$$

$$= n u^{n-1} \cdot u'(x)$$

$$= n u(x)^{n-1} \cdot \underline{\underline{u'(x)}}$$

Chain Rule



$$\frac{d}{dx} u(x)^n = n \cdot u(x)^{n-1} \cdot u'(x) \quad \text{So } \downarrow$$

$$\int \underbrace{u'(x) \cdot u(x)^n}_{\substack{\text{derivative of this} \\ \text{is equal to} \\ \text{this}}} dx = \boxed{\frac{u(x)^{n+1}}{n+1} + C}$$

Question: why is $u(x)$ not in answer?

$$\underline{2 = 2 \cdot 2 \cdot 2 = 8}$$

there is no 3

$$\sqrt[3]{8} = 2$$

in answer

Similar to this

$$\int u(x)^n \cdot u'(x) dx = \frac{u(x)^{n+1}}{n+1} + C$$

take derivative of all

$$\frac{d}{dx} \left[\frac{1}{(n+1)} u(x)^{n+1} \right] = \frac{1}{(n+1)} \cdot \frac{d}{dx} \left[u(x)^{n+1} \right]$$

no $u'(x)$ \rightarrow

$$= \frac{1}{(n+1)} \cdot (n+1) \cdot u(x)^n \cdot \frac{du}{dx}$$

$$= u(x)^n \cdot u'(x)$$

\leftarrow yes $u'(x)$
