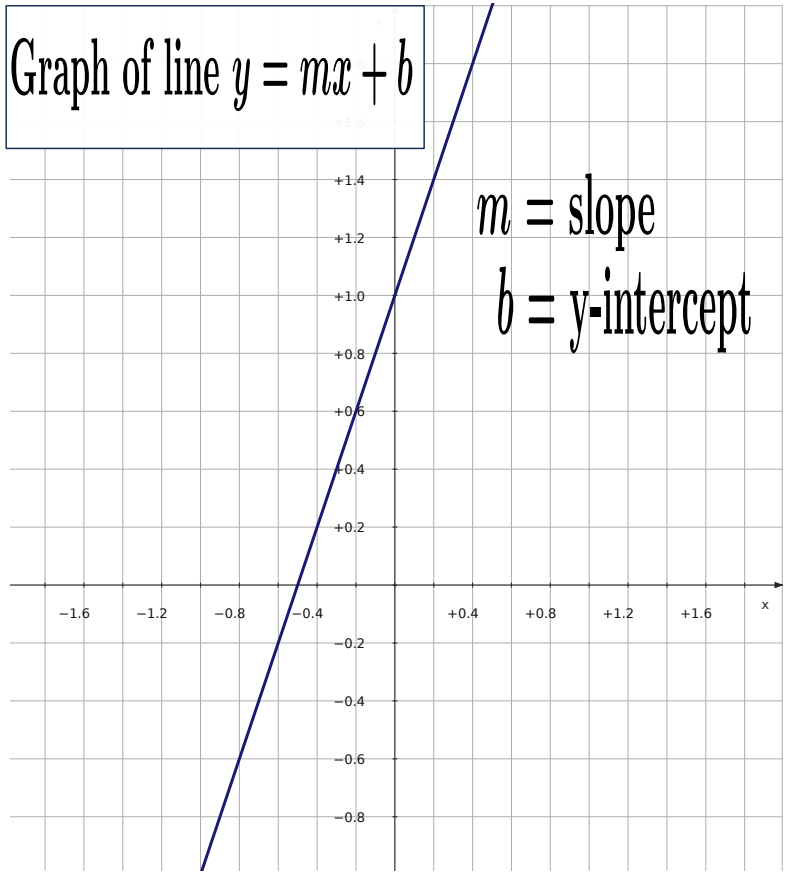
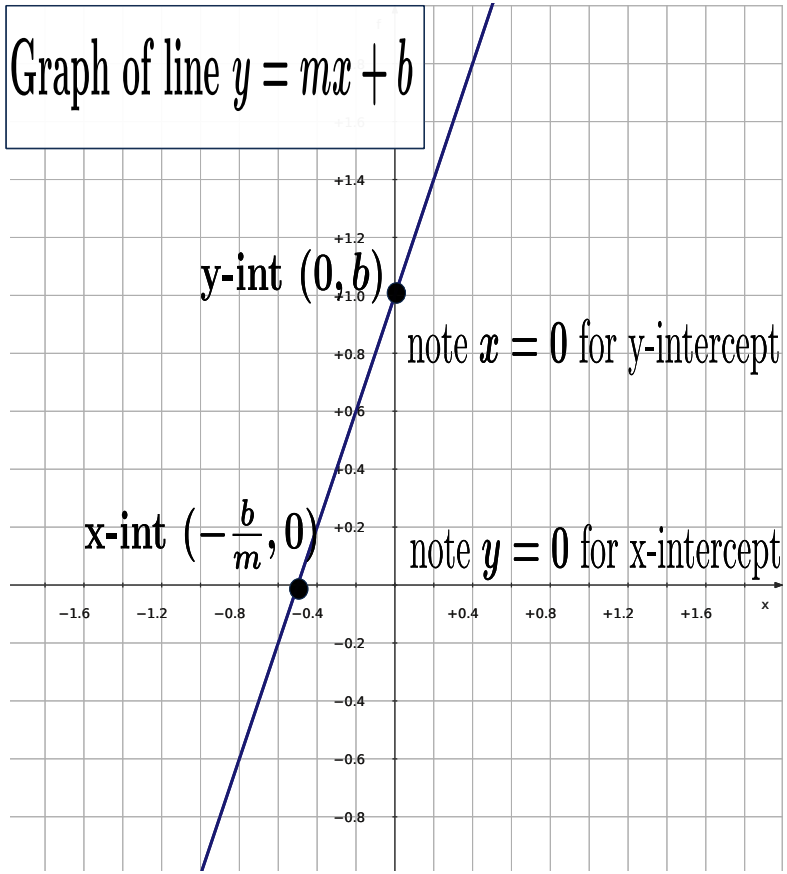
Linear Functions

Graph of Line



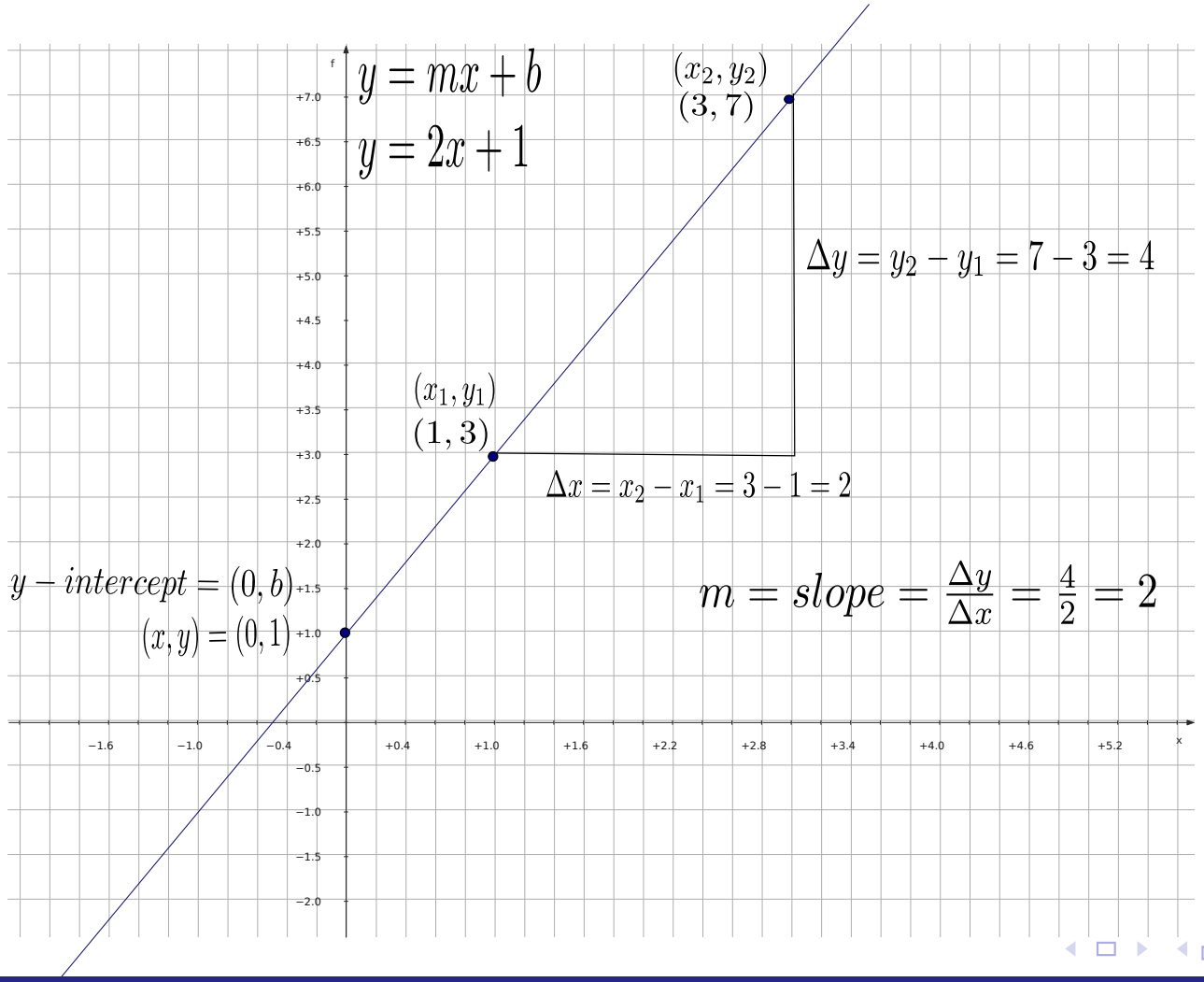
Linear Functions

x and y intercepts



Slope

$\mathbf{m = \frac{rise}{run}}$



What is a slope?

definition

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Slopes of Lines

increase and decrease

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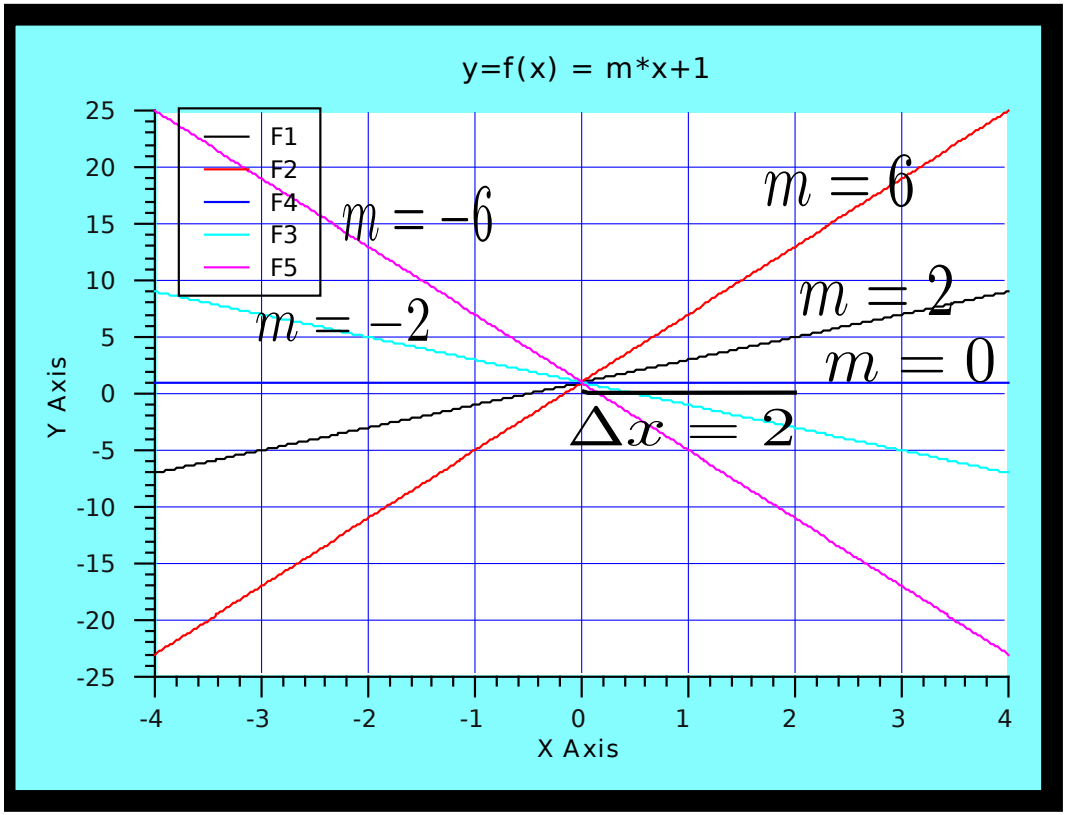

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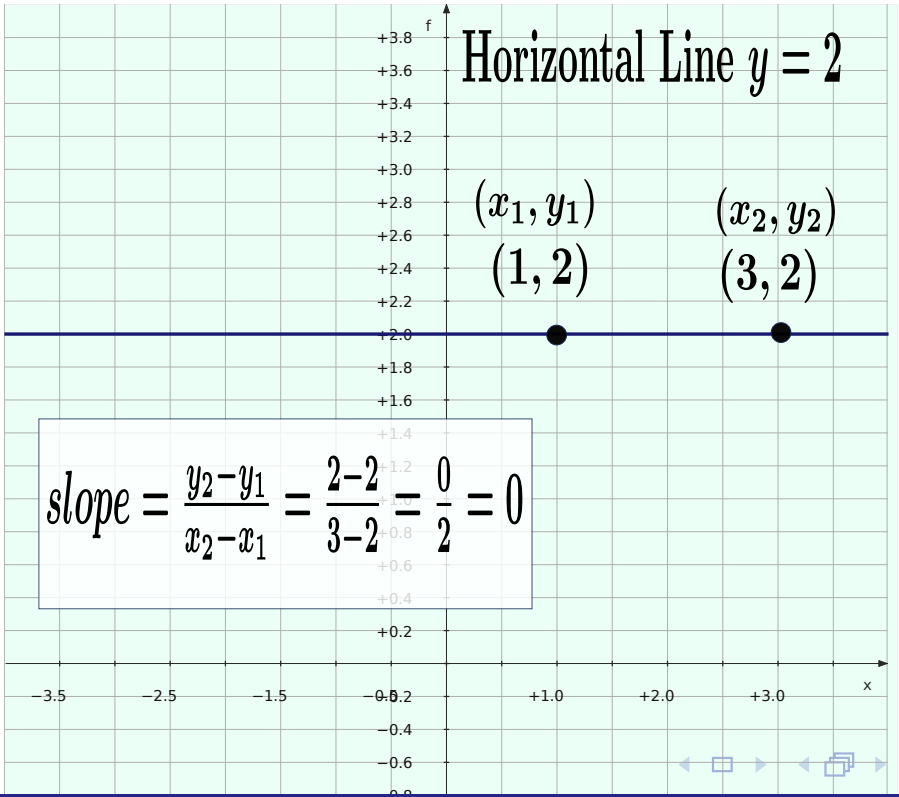

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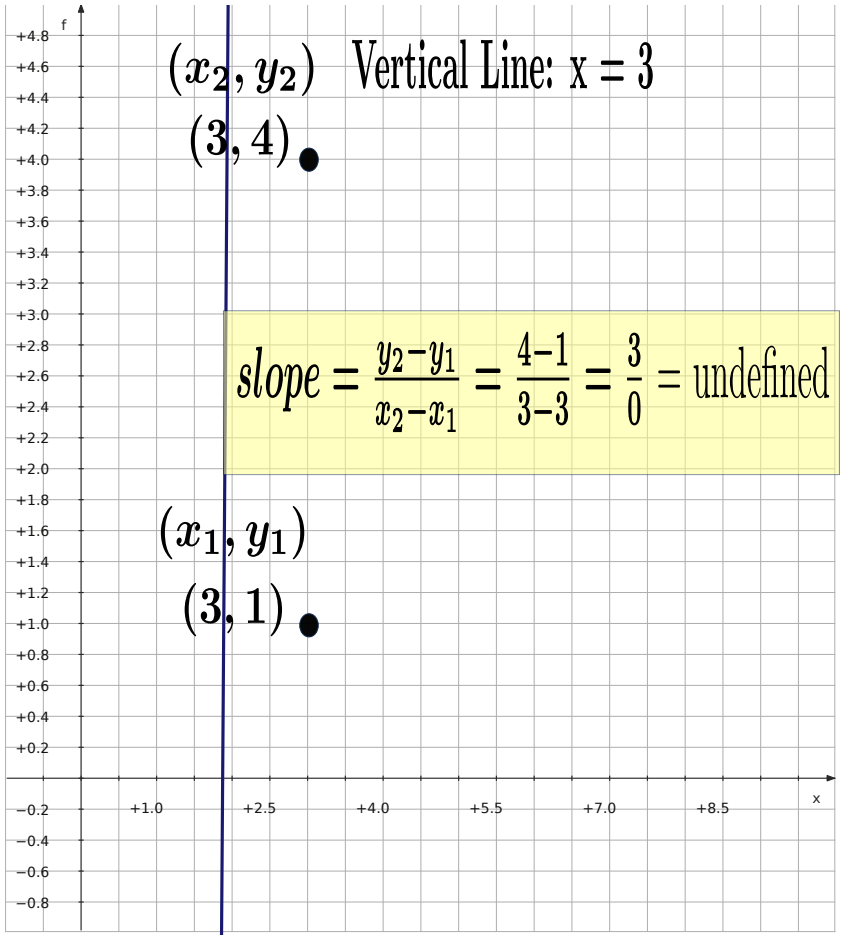
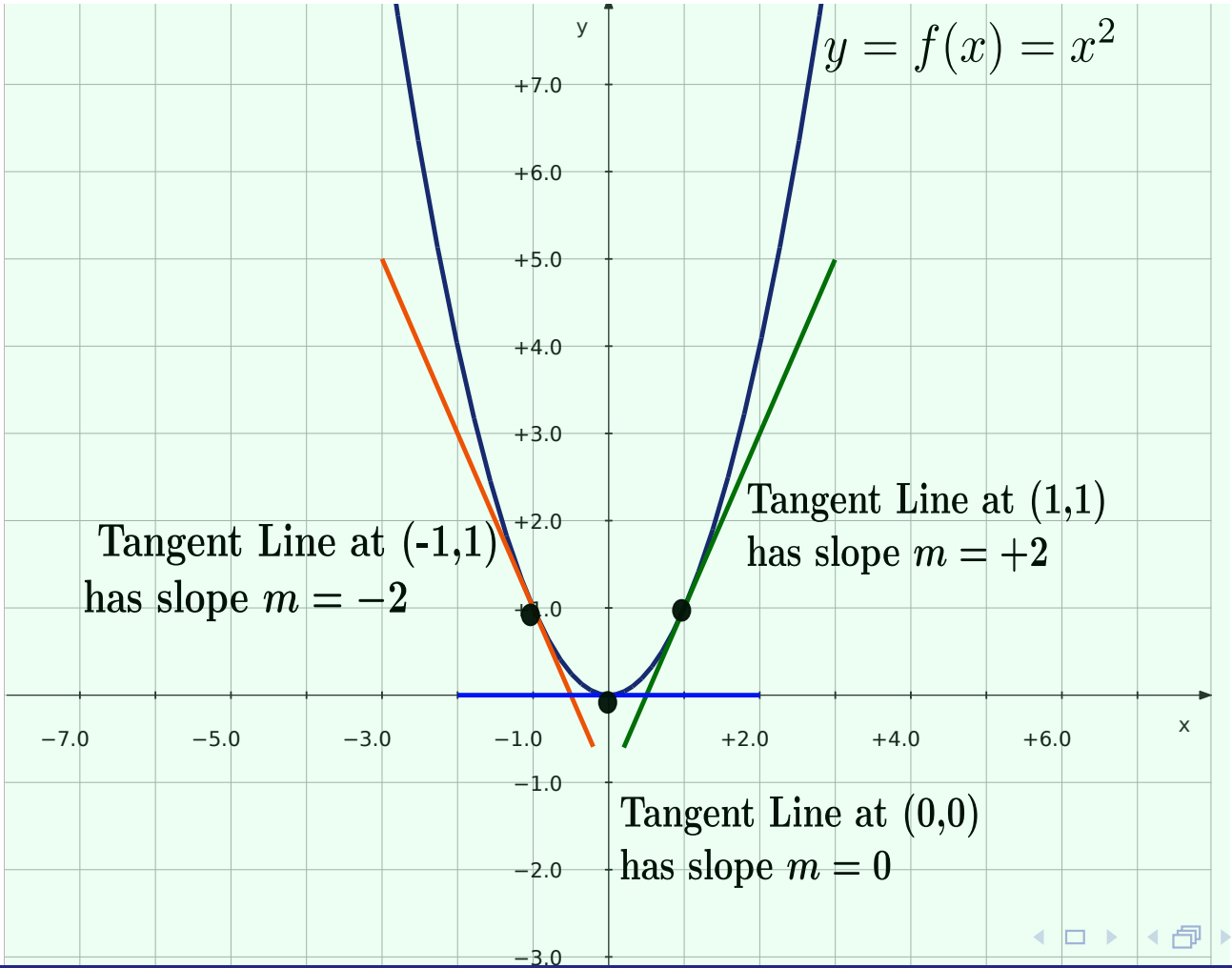


Tangent Lines

from algebra to calculus



Equations of lines

Point-Slope Equation

Point-Slope Equation of Line, $\mathbf{y - y_0 = m(x - x_0)}$

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   - final expression in **slope-intercept form**