

Marginal Analysis

Math165: Business Calculus

Roy M. Lowman

Spring 2010

Marginal Analysis

Marginal Cost - two definitions

Marginal cost: From Wikipedia, the free encyclopedia
In economics and finance, marginal cost is the change in total cost that arises when the quantity produced changes by one unit. That is, it is the cost of producing one more unit of a good. Mathematically, the marginal cost (MC) function is expressed as the first derivative of the total cost (TC) function with respect to quantity (Q).

Note that there are two definitions:

- **Practical Definition:** marginal cost is the change in total cost that arises when the quantity produced changes by one unit
- **Formal definition used in calculus:** marginal cost (MC) function is expressed as the first derivative of the total cost (TC) function with respect to quantity (q).

Marginal Analysis

Marginal Cost - two definitions

Marginal cost: From Wikipedia, the free encyclopedia

In economics and finance, marginal cost is the change in total cost that arises when the quantity produced changes by one unit. That is, it is the cost of producing one more unit of a good.

Mathematically, the marginal cost (MC) function is expressed as the first derivative of the total cost (TC) function with respect to quantity (Q).

Note that there are two definitions:

- **Practical Definition:** marginal cost is the change in total cost that arises when the quantity produced changes by one unit
- **Formal definition used in calculus:** marginal cost (MC) function is expressed as the first derivative of the total cost (TC) function with respect to quantity (q).

Marginal Analysis

Marginal Cost - two definitions

Marginal cost: From Wikipedia, the free encyclopedia
In economics and finance, marginal cost is the change in total cost that arises when the quantity produced changes by one unit. That is, it is the cost of producing one more unit of a good. Mathematically, the marginal cost (MC) function is expressed as the first derivative of the total cost (TC) function with respect to quantity (Q).

Note that there are two definitions:

- **Practical Definition:** marginal cost is the change in total cost that arises when the quantity produced changes by one unit
- **Formal definition used in calculus:** marginal cost (MC) function is expressed as the first derivative of the total cost (TC) function with respect to quantity (q).

Marginal Analysis

Marginal Cost - two definitions

Marginal cost: From Wikipedia, the free encyclopedia
In economics and finance, marginal cost is the change in total cost that arises when the quantity produced changes by one unit. That is, it is the cost of producing one more unit of a good. Mathematically, the marginal cost (MC) function is expressed as the first derivative of the total cost (TC) function with respect to quantity (Q).

Note that there are two definitions:

- **Practical Definition:** marginal cost is the change in total cost that arises when the quantity produced changes by one unit
- **Formal definition used in calculus:** marginal cost (MC) function is expressed as the first derivative of the total cost (TC) function with respect to quantity (q).

Marginal Analysis

Marginal Cost - two definitions

Marginal cost: From Wikipedia, the free encyclopedia
In economics and finance, marginal cost is the change in total cost that arises when the quantity produced changes by one unit. That is, it is the cost of producing one more unit of a good. Mathematically, the marginal cost (MC) function is expressed as the first derivative of the total cost (TC) function with respect to quantity (Q).

Note that there are two definitions:

- **Practical Definition:** marginal cost is the change in total cost that arises when the quantity produced changes by one unit
- **Formal definition used in calculus:** marginal cost (MC) function is expressed as the first derivative of the total cost (TC) function with respect to quantity (q).

Marginal Analysis

Marginal Cost - two definitions

Marginal cost: From Wikipedia, the free encyclopedia
In economics and finance, marginal cost is the change in total cost that arises when the quantity produced changes by one unit. That is, it is the cost of producing one more unit of a good. Mathematically, the marginal cost (MC) function is expressed as the first derivative of the total cost (TC) function with respect to quantity (Q).

Note that there are two definitions:

- **Practical Definition:** marginal cost is the change in total cost that arises when the quantity produced changes by one unit
- **Formal definition used in calculus:** marginal cost (MC) function is expressed as the first derivative of the total cost (TC) function with respect to quantity (q).

Marginal Analysis

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

q is quantity produced or production level

p is price per unit (amount consumer pays for one)

If all are functions of **q**

Cost Function: $C(q)$ is cost to produce q units

Revenue Function: $R(q)$ is income from selling q units

Profit Function: $P(q) = R(q) - C(q)$

Profit = Revenue - Cost (common sense)

Marginal Cost: $MC = \frac{dP}{dq}$, slope of cost function

Marginal Revenue: $MR = \frac{dR}{dq}$, slope of revenue function

Marginal Profit: $MP = \frac{dP}{dq}$, slope of profit function

Marginal Analysis

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

q is quantity produced or production level

p is price per unit (amount consumer pays for one)

If all are functions of **q**

Cost Function: $C(q)$ is cost to produce q units

Revenue Function: $R(q)$ is income from selling q units

Profit Function: $P(q) = R(q) - C(q)$

Profit = Revenue - Cost (common sense)

Marginal Cost: $MC = \frac{dP}{dq}$, slope of cost function

Marginal Revenue: $MR = \frac{dR}{dq}$, slope of revenue function

Marginal Profit: $MP = \frac{dP}{dq}$, slope of profit function

Marginal Analysis

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

q is quantity produced or production level

p is price per unit (amount consumer pays for one)

If all are functions of **q**

Cost Function: $C(q)$ is cost to produce q units

Revenue Function: $R(q)$ is income from selling q units

Profit Function: $P(q) = R(q) - C(q)$

Profit = Revenue - Cost (common sense)

Marginal Cost: $MC = \frac{dP}{dq}$, slope of cost function

Marginal Revenue: $MR = \frac{dR}{dq}$, slope of revenue function

Marginal Profit: $MP = \frac{dP}{dq}$, slope of profit function

Marginal Analysis

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

q is quantity produced or production level

p is price per unit (amount consumer pays for one)

If all are functions of **q**

Cost Function: $C(q)$ is cost to produce **q** units

Revenue Function: $R(q)$ is income from selling **q** units

Profit Function: $P(q) = R(q) - C(q)$

Profit = Revenue - Cost (common sense)

Marginal Cost: $MC = \frac{dP}{dq}$, slope of cost function

Marginal Revenue: $MR = \frac{dR}{dq}$, slope of revenue function

Marginal Profit: $MP = \frac{dP}{dq}$, slope of profit function

Marginal Analysis

Profit = Revenue - Cost

Profit = Revenue - Cost

q is quantity produced or production level

p is price per unit (amount consumer pays for one)

If all are functions of **q**

Cost Function: **C(q)** is cost to produce **q** units

Revenue Function: **R(q)** is income from selling **q** units

Profit Function: **P(q) = R(q) - C(q)**

Profit = Revenue - Cost (common sense)

Marginal Cost: **MC** = $\frac{dP}{dq}$, slope of cost function

Marginal Revenue: **MR** = $\frac{dR}{dq}$, slope of revenue function

Marginal Profit: **MP** = $\frac{dP}{dq}$, slope of profit function

Marginal Analysis

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

q is quantity produced or production level

p is price per unit (amount consumer pays for one)

If all are functions of **q**

Cost Function: $C(q)$ is cost to produce **q** units

Revenue Function: $R(q)$ is income from selling **q** units

Profit Function: $P(q) = R(q) - C(q)$

Profit = Revenue - Cost (common sense)

Marginal Cost: $MC = \frac{dC}{dq}$, slope of cost function

Marginal Revenue: $MR = \frac{dR}{dq}$, slope of revenue function

Marginal Profit: $MP = \frac{dP}{dq}$, slope of profit function

Marginal Analysis

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

q is quantity produced or production level

p is price per unit (amount consumer pays for one)

If all are functions of **q**

Cost Function: $C(q)$ is cost to produce **q** units

Revenue Function: $R(q)$ is income from selling **q** units

Profit Function: $P(q) = R(q) - C(q)$

Profit = Revenue - Cost (common sense)

Marginal Cost: $MC = \frac{dP}{dq}$, slope of cost function

Marginal Revenue: $MR = \frac{dR}{dq}$, slope of revenue function

Marginal Profit: $MP = \frac{dP}{dq}$, slope of profit function

Marginal Analysis

Profit = Revenue - Cost

Profit = Revenue - Cost

q is quantity produced or production level

p is price per unit (amount consumer pays for one)

If all are functions of **q**

Cost Function: $C(q)$ is cost to produce **q** units

Revenue Function: $R(q)$ is income from selling **q** units

Profit Function: $P(q) = R(q) - C(q)$

Profit = Revenue - Cost (common sense)

Marginal Cost: $MC = \frac{dC}{dq}$, slope of cost function

Marginal Revenue: $MR = \frac{dR}{dq}$, slope of revenue function

Marginal Profit: $MP = \frac{dP}{dq}$, slope of profit function

Marginal Analysis

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

q is quantity produced or production level

p is price per unit (amount consumer pays for one)

If all are functions of **q**

Cost Function: $C(q)$ is cost to produce **q** units

Revenue Function: $R(q)$ is income from selling **q** units

Profit Function: $P(q) = R(q) - C(q)$

Profit = Revenue - Cost (common sense)

Marginal Cost: $MC = \frac{dC}{dq}$, slope of cost function

Marginal Revenue: $MR = \frac{dR}{dq}$, slope of revenue function

Marginal Profit: $MP = \frac{dP}{dq}$, slope of profit function

Marginal Analysis

definitions

Definition (Marginal Cost)

$$MC = \frac{dC}{dq} = \lim_{\Delta q \rightarrow 0} \left[\frac{C(q + \Delta q) - C(q)}{\Delta q} \right]$$

- Cost functions are often found by using statistical methods to find a continuous function that best fits the data.
- q is treated as a continuous real number and the above limit exists, the marginal cost is the slope of the cost function. This makes sense when q can be large.
- Marginal analysis is often done using real data and not statistical functions. In this case the above limit does not exist!

Definition (Marginal Cost)

$$MC = \frac{dC}{dq} = \lim_{\Delta q \rightarrow 0} \left[\frac{C(q + \Delta q) - C(q)}{\Delta q} \right]$$

- Cost functions are often found by using statistical methods to find a continuous function that best fits the data.
- **q** is treated as a continuous real number and the above limit exists, the marginal cost is the slope of the cost function. This makes sense when **q** can be large.
- Marginal analysis is often done using real data and not statistical functions. In this case the above limit does not exist!

Definition (Marginal Cost)

$$MC = \frac{dC}{dq} = \lim_{\Delta q \rightarrow 0} \left[\frac{C(q + \Delta q) - C(q)}{\Delta q} \right]$$

- Cost functions are often found by using statistical methods to find a continuous function that best fits the data.
- q is treated as a continuous real number and the above limit exists, the marginal cost is the slope of the cost function. This makes sense when q can be large.
- Marginal analysis is often done using real data and not statistical functions. In this case the above limit does not exist!

Definition (Marginal Cost)

$$MC = \frac{dC}{dq} = \lim_{\Delta q \rightarrow 0} \left[\frac{C(q + \Delta q) - C(q)}{\Delta q} \right]$$

- Cost functions are often found by using statistical methods to find a continuous function that best fits the data.
- q is treated as a continuous real number and the above limit exists, the marginal cost is the slope of the cost function. This makes sense when q can be large.
- Marginal analysis is often done using real data and not statistical functions. In this case the above limit does not exist!

Marginal Analysis

definitions

When doing marginal analysis with real data this limit does not exist:

$$MC = \frac{dC}{dq} = \lim_{\Delta q \rightarrow 0} \left[\frac{C(q + \Delta q) - C(q)}{\Delta q} \right]$$

Here is an explanation:

- q is the number of items produced, it must be a non-negative integer. You can not produce a fraction of an item.
- Possible values of q are $0, 1, 2, 3, 4, 5, 6, 7, 8, \dots$,
- So possible values of Δq are also $0, 1, 2, 3, 4, 5, 6, 7, 8, \dots$,
- For example if $q = 4$ then $\Delta q = 5 - 4 = 1$, if $q_2 = 8, q_1 = 6$ then $\Delta q = 8 - 6 = 2$.
- Δq can not go to zero as $.1, .01, .0001, \dots$ so the limit does not exist.
- The best that can be done is to estimate the limit for MC by using the smallest possible value of Δq which is $\Delta q = 1$

Marginal Analysis

definitions

When doing marginal analysis with real data this limit does not exist:

$$MC = \frac{dC}{dq} = \lim_{\Delta q \rightarrow 0} \left[\frac{C(q + \Delta q) - C(q)}{\Delta q} \right]$$

Here is an explanation:

- q is the number of items produced, it must be a non-negative integer. You can not produce a fraction of an item.
- Possible values of q are $0, 1, 2, 3, 4, 5, 6, 7, 8, \dots$,
- So possible values of Δq are also $0, 1, 2, 3, 4, 5, 6, 7, 8, \dots$,
- For example if $q = 4$ then $\Delta q = 5 - 4 = 1$, if $q_2 = 8, q_1 = 6$ then $\Delta q = 8 - 6 = 2$.
- Δq can not go to zero as $.1, .01, .0001, \dots$ so the limit does not exist.
- The best that can be done is to estimate the limit for MC by using the smallest possible value of Δq which is $\Delta q = 1$

Marginal Analysis

definitions

When doing marginal analysis with real data this limit does not exist:

$$MC = \frac{dC}{dq} = \lim_{\Delta q \rightarrow 0} \left[\frac{C(q + \Delta q) - C(q)}{\Delta q} \right]$$

Here is an explanation:

- **q** is the number of items produced, it must be a non-negative integer. You can not produce a fraction of an item.
- Possible values of **q** are 0, 1, 2, 3, 4, 5, 6, 7, 8, ... ,
- So possible values of **Δq** are also 0, 1, 2, 3, 4, 5, 6, 7, 8, ... ,
- For example if **q** = 4 then **Δq** = 5 - 4 = 1, if **q**₂ = 8, **q**₁ = 6 then **Δq** = 8 - 6 = 2.
- **Δq** can not go to zero as .1, .01, .0001, ... so the limit does not exist.
- The best that can be done is to estimate the limit for MC by using the smallest possible value of **Δq** which is **Δq** = 1

Marginal Analysis

definitions

When doing marginal analysis with real data this limit does not exist:

$$MC = \frac{dC}{dq} = \lim_{\Delta q \rightarrow 0} \left[\frac{C(q + \Delta q) - C(q)}{\Delta q} \right]$$

Here is an explanation:

- q is the number of items produced, it must be a non-negative integer. You can not produce a fraction of an item.
- Possible values of q are **0, 1, 2, 3, 4, 5, 6, 7, 8, \dots**,
- So possible values of Δq are also **0, 1, 2, 3, 4, 5, 6, 7, 8, \dots**,
- For example if $q = 4$ then $\Delta q = 5 - 4 = 1$, if $q_2 = 8, q_1 = 6$ then $\Delta q = 8 - 6 = 2$.
- Δq can not go to zero as **.1, .01, .0001, \dots** so the limit does not exist.
- The best that can be done is to estimate the limit for MC by using the smallest possible value of Δq which is $\Delta q = 1$

Marginal Analysis

definitions

When doing marginal analysis with real data this limit does not exist:

$$MC = \frac{dC}{dq} = \lim_{\Delta q \rightarrow 0} \left[\frac{C(q + \Delta q) - C(q)}{\Delta q} \right]$$

Here is an explanation:

- q is the number of items produced, it must be a non-negative integer. You can not produce a fraction of an item.
- Possible values of q are **0, 1, 2, 3, 4, 5, 6, 7, 8, ...**,
- So possible values of Δq are also **0, 1, 2, 3, 4, 5, 6, 7, 8, ...**,
- For example if $q = 4$ then $\Delta q = 5 - 4 = 1$, if $q_2 = 8, q_1 = 6$ then $\Delta q = 8 - 6 = 2$.
- Δq can not go to zero as **.1, .01, .0001, ...** so the limit does not exist.
- The best that can be done is to estimate the limit for MC by using the smallest possible value of Δq which is $\Delta q = 1$

Marginal Analysis

definitions

When doing marginal analysis with real data this limit does not exist:

$$MC = \frac{dC}{dq} = \lim_{\Delta q \rightarrow 0} \left[\frac{C(q + \Delta q) - C(q)}{\Delta q} \right]$$

Here is an explanation:

- q is the number of items produced, it must be a non-negative integer. You can not produce a fraction of an item.
- Possible values of q are $0, 1, 2, 3, 4, 5, 6, 7, 8, \dots$,
- So possible values of Δq are also $0, 1, 2, 3, 4, 5, 6, 7, 8, \dots$,
- For example if $q = 4$ then $\Delta q = 5 - 4 = 1$, if $q_2 = 8, q_1 = 6$ then $\Delta q = 8 - 6 = 2$.
- Δq can not go to zero as $.1, .01, .0001, \dots$ so the limit does not exist.
- The best that can be done is to estimate the limit for MC by using the smallest possible value of Δq which is $\Delta q = 1$

Marginal Analysis

definitions

When doing marginal analysis with real data this limit does not exist:

$$MC = \frac{dC}{dq} = \lim_{\Delta q \rightarrow 0} \left[\frac{C(q + \Delta q) - C(q)}{\Delta q} \right]$$

Here is an explanation:

- q is the number of items produced, it must be a non-negative integer. You can not produce a fraction of an item.
- Possible values of q are $0, 1, 2, 3, 4, 5, 6, 7, 8, \dots$,
- So possible values of Δq are also $0, 1, 2, 3, 4, 5, 6, 7, 8, \dots$,
- For example if $q = 4$ then $\Delta q = 5 - 4 = 1$, if $q_2 = 8, q_1 = 6$ then $\Delta q = 8 - 6 = 2$.
- Δq can not go to zero as **.1, .01, .0001, ...** so the limit does not exist.
- The best that can be done is to estimate the limit for MC by using the smallest possible value of Δq which is $\Delta q = 1$

Marginal Analysis

definitions

When doing marginal analysis with real data this limit does not exist:

$$MC = \frac{dC}{dq} = \lim_{\Delta q \rightarrow 0} \left[\frac{C(q + \Delta q) - C(q)}{\Delta q} \right]$$

Here is an explanation:

- q is the number of items produced, it must be a non-negative integer. You can not produce a fraction of an item.
- Possible values of q are $0, 1, 2, 3, 4, 5, 6, 7, 8, \dots$,
- So possible values of Δq are also $0, 1, 2, 3, 4, 5, 6, 7, 8, \dots$,
- For example if $q = 4$ then $\Delta q = 5 - 4 = 1$, if $q_2 = 8, q_1 = 6$ then $\Delta q = 8 - 6 = 2$.
- Δq can not go to zero as $.1, .01, .0001, \dots$ so the limit does not exist.
- The best that can be done is to estimate the limit for MC by using the smallest possible value of Δq which is $\Delta q = 1$

Marginal Analysis

definitions

When doing marginal analysis with real data this limit does not exist:

$$MC = \frac{dC}{dq} = \lim_{\Delta q \rightarrow 0} \left[\frac{C(q + \Delta q) - C(q)}{\Delta q} \right]$$

Here is an explanation:

- q is the number of items produced, it must be a non-negative integer. You can not produce a fraction of an item.
- Possible values of q are $0, 1, 2, 3, 4, 5, 6, 7, 8, \dots$,
- So possible values of Δq are also $0, 1, 2, 3, 4, 5, 6, 7, 8, \dots$,
- For example if $q = 4$ then $\Delta q = 5 - 4 = 1$, if $q_2 = 8, q_1 = 6$ then $\Delta q = 8 - 6 = 2$.
- Δq can not go to zero as $.1, .01, .0001, \dots$ so the limit does not exist.
- The best that can be done is to estimate the limit for MC by using the smallest possible value of Δq which is $\Delta q = 1$

Marginal Analysis

definitions

The best that can be done is to estimate the limit for **MC** by using the smallest possible value of Δq which is $\Delta q = 1$

Definition (Marginal Cost Approximation)

$$MC = \frac{dC}{dq} \approx \left[\frac{C(q+1) - C(q)}{1} \right] = C(q+1) - C(q)$$

Interpretation: if the current production is now q then the marginal cost is the cost to produce one more item. This is often used as the definition of MC and $\frac{dC}{dq}$ can be used to estimate the cost of producing the $q+1$ th item if q is the current production level.

Example

If $q = 100$, and $C(q) = 100 + 6q^2$ then $\frac{dC}{dq} = 12q$ and the cost of producing the 101th item is $\$12(100) = \1200

Marginal Analysis

definitions

The best that can be done is to estimate the limit for **MC** by using the smallest possible value of Δq which is $\Delta q = 1$

Definition (Marginal Cost Approximation)

$$MC = \frac{dC}{dq} \approx \left[\frac{C(q+1) - C(q)}{1} \right] = C(q+1) - C(q)$$

Interpretation: if the current production is now q then the marginal cost is the cost to produce one more item. This is often used as the definition of MC and $\frac{dC}{dq}$ can be used to estimate the cost of producing the $q+1$ th item if q is the current production level.

Example

If $q = 100$, and $C(q) = 100 + 6q^2$ then $\frac{dC}{dq} = 12q$ and the cost of producing the 101th item is $\$12(100) = \1200

Marginal Analysis

definitions

The best that can be done is to estimate the limit for **MC** by using the smallest possible value of Δq which is $\Delta q = 1$

Definition (Marginal Cost Approximation)

$$MC = \frac{dC}{dq} \approx \left[\frac{C(q+1) - C(q)}{1} \right] = C(q+1) - C(q)$$

Interpretation: if the current production is now q then the marginal cost is the cost to produce one more item. This is often used as the definition of MC and $\frac{dC}{dq}$ can be used to estimate the cost of producing the $q + 1$ th item if q is the current production level.

Example

If $q = 100$, and $C(q) = 100 + 6q^2$ then $\frac{dC}{dq} = 12q$ and the cost of producing the 101th item is $\$12(100) = \1200

Marginal Analysis

MC, MR, MP

Similar definitions apply for Marginal Revenue and Marginal Profit.

Definition

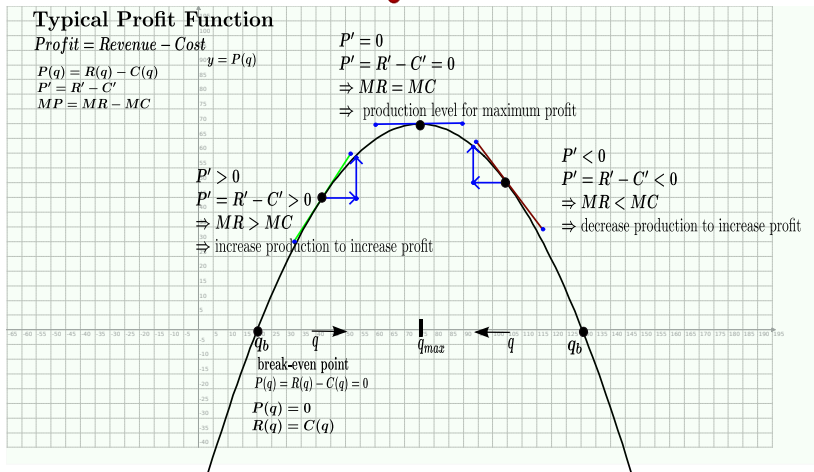
$$MC = \frac{dC}{dq} = \lim_{\Delta q \rightarrow 0} \left[\frac{C(q + \Delta q) - C(q)}{\Delta q} \right] \approx C(q + 1) - C(q)$$

$$MR = \frac{dR}{dq} = \lim_{\Delta q \rightarrow 0} \left[\frac{R(q + \Delta q) - R(q)}{\Delta q} \right] \approx R(q + 1) - R(q)$$

$$MP = \frac{dP}{dq} = \lim_{\Delta q \rightarrow 0} \left[\frac{P(q + \Delta q) - P(q)}{\Delta q} \right] \approx P(q + 1) - P(q)$$

Marginal Analysis

making decisions



Marginal Analysis

making decisions with data

MR > MC increase production to increase profit

MR = MC production level gives maximum profit, do not change production level

MR < MC decrease production level to increase profit

Example (Given Profit Data)

q	P(q)	$\Delta P(q) = P(q + 1) - P(q)$
100	25	28-25 = +3
101	28	+2
102	30	0
103	30	-1
104	29	-2
105	27	-

If the current production level is **q = 104** the marginal profit is negative so the decision should be to decrease production to increase profit.