Math 165, Fall 2010, Lowman

L11 - W4L3 Friday

Marginal Analysis Example

Profit = Revenue − Cost

$$P(q) = R(q) - C(q)$$

Variable Names:

P = profit function

R = revenue function

C = cost function

c = cost per unit to producer/seller

p = price per unit to consumer/buyer

In addition, a demand relation is needed to relate p and q.

This is often given as $p = D(q)$

or $q = D(p)$

Simple Example:

given $c = \overset{8}{6}$, cost per unit.

$q = 100 - 2 \cdot p$, demand

Find p_{max} the price that must be charged to get maximum profit.

Also find q_{max} the quantity that must be made and sold at p_{max}.

Cost Function: assume that all cost is from c.

$$C(q) = c \cdot q$$

$$\boxed{C(q) = 6 \cdot q}$$

Revenue Function:

$R(q,p) = p \cdot q$, this is correct but need $R(q)$ and not $R(q,p)$.

Use the demand function to eliminate p.

$q = 100 - 2p$ · first solve for p in terms of q.

$$q = 100 - 2\cdot\cancel{p}$$
$$+2p \qquad\qquad +\cancel{2p}$$

$$2p + \cancel{q} = 100$$
$$-\cancel{q} \qquad\qquad -q$$

$$\cancel{\tfrac{1}{2}}(\cancel{2}^{1}p) = \tfrac{1}{2}(100 - q)$$

$$p = \tfrac{1}{2}\cdot 100 - \tfrac{1}{2}\cdot q$$

$$\boxed{p = 50 - \frac{q}{2}}$$

this is the same demand relation but turned inside-out.

Now use to replace p in $R(q,p)$.

$$R = p \cdot q$$
$$= \left(50 - \frac{q}{2}\right)\cdot q$$

$$\boxed{R(q) = 50q - \frac{q^2}{2}}$$

now find Profit function

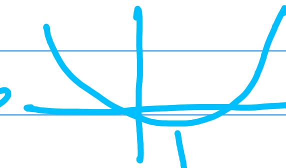
Profit Function:

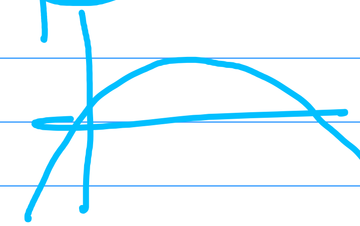
$$P(q) = R(q) - C(q)$$

$$P(q) = \left(50q - \frac{q^2}{2}\right) - (6 \cdot q)$$

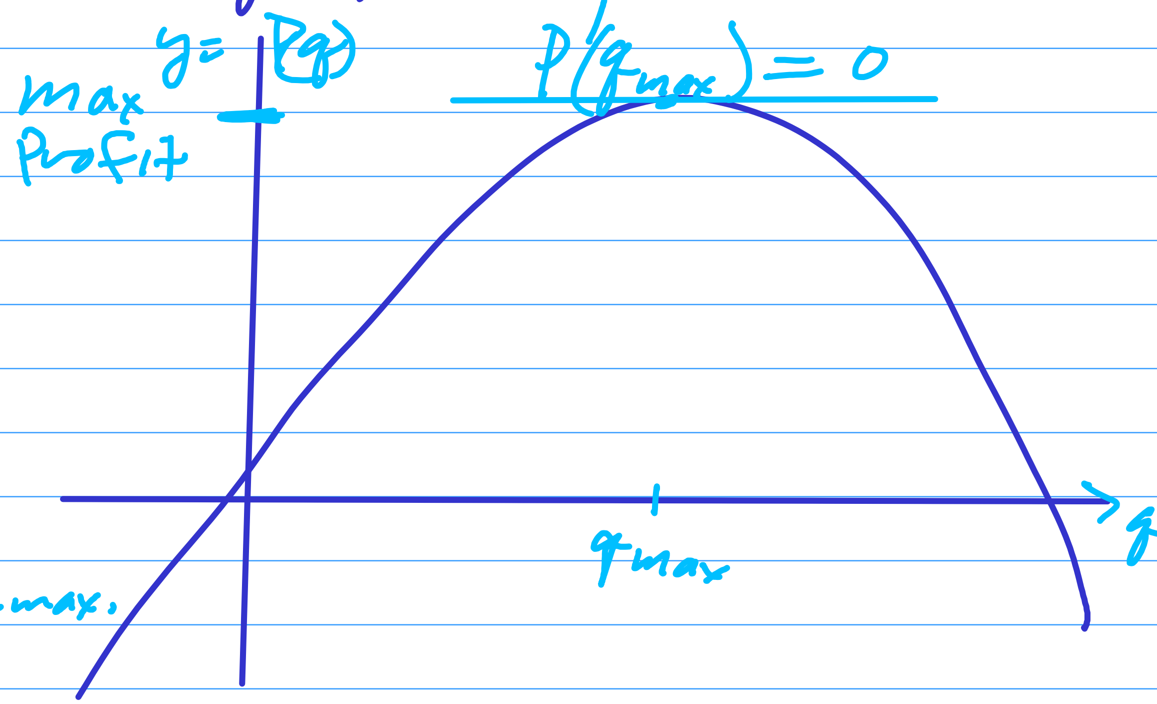
$$\boxed{P(q) = -\frac{q^2}{2} + 44q}$$

note all functions of form $P(x) = ax^2 + bx + c$ will graph as a parabola.

- if a = positive then opens up 

- if a = negative then opens down 

In this case $P(q)$ is a quadratic function that opens down. Therefore $P(q)$ has a maximum value and q_{max} can be found.



Observe that $P(q)$ is a maximum when it's slope is zero.

So find q_{max} by

Setting $P'(q) = 0$ and solving for $q = q_{max}$.

$$P(q) = -\frac{q^2}{2} + 44q$$

$$\frac{d}{dq}\left(P(q)\right) = \frac{d}{dq}\left(-\frac{q^2}{2} + 44q\right)$$

$$P'(q) = \frac{d}{dq}\left(-\frac{1}{2}\cdot q^2\right) + \frac{d}{dq}(44q)$$

$$P'(q) = -\frac{1}{2}\frac{d}{dq}(q^2) + 44\cancel{\frac{d}{dq}(q)}^{1}$$

$$= \cancel{-\frac{1}{2}\cdot 2}q + 44$$

$$P'(q) = -q + 44$$, now set $P' = 0$ and solve for q_{max}.

$$-q_{max} + 44 = 0$$

$$\boxed{q_{max} = 44}$$

This is the quantity that must be made and sold to make a maximum profit.

Now use the demand relation to find p_{max}

$$p = 50 - \frac{q}{2} \qquad \text{so}$$

$$p_{max} = 50 - (44)/2$$

$$p_{max} = 50 - 22$$

$$\boxed{p_{max} = \$28}$$

this is the price/unit that you must charge to get Maximum Profit.

Now, what is the maximum Profit?

$$P_{max} = P(q_{max}) = P(44)$$

$$= -\frac{q^2}{2} + 44q \Big|_{q=44}$$

$$= q\left(44 - \frac{q}{2}\right)\Big|_{q=44}$$

$$= 44\left(44 - \frac{44}{2}\right)$$

$$= 44(44 - 22)$$

$$= 44(22)$$

$$\boxed{P_{max} = \$968.00 \quad \text{Max Profit}}$$