Math 165 Fall 2010 Lowman
Week 5 Lec 3

- Review Derivative Rules
- Odd topics from Chapter 2
- Chapter 5 next

Review:

$$\frac{d}{dx} f(x)^n = n \cdot f(x)^{n-1} \cdot f'(x)$$ Power Rule

$$\frac{d}{dx} e^{f(x)} = e^{f(x)} \cdot f'(x)$$ Exponential Rule

$$\frac{d}{dx} \ln f(x) = \frac{f'(x)}{f(x)}$$ Log Rule

ex $$\frac{d}{dx}(2x^3+x)^5 = 5\cdot(2x^3+x)^4 \cdot \frac{d}{dx}(2x^3+x)$$
$$= 5\cdot(2x^3+x)^4\cdot(6x^2+1)$$
$$= 5\cdot(6x^2+1)\cdot(2x^3+x)^4$$

ex. $$\frac{d}{dx} x^3 = 3\cdot x^2 \cdot \cancel{\frac{dx}{dx}}^{\,1} = 3x^2$$ same as for simple rule ex. $\frac{d}{dx} e^{4x^3+x^2} = e^{4x^3+x^2} \cdot \frac{d}{dx}(4x^3+x^2)$

$= e^{4x^3+x^2} \cdot (12x^2+2x)$

$= 2x\cdot(6x+1)\cdot e^{4x^3+x^2}$

ex. $\frac{d}{dx} e^x = e^x \cdot \cancel{\frac{dx}{dx}}^{\,1} = e^x$ same as for simple version of exponential rule.

ex. $\frac{d}{dx} \ln(3x^5+7x) = \dfrac{\frac{d}{dx}(3x^5+7x)}{(3x^5+7x)}$

$= \dfrac{15x^4+7}{3x^5+7x}$

ex $\frac{d}{dx} \ln x = \dfrac{\cancel{\frac{d}{dx}x}^{\,1}}{x} = \dfrac{1}{x}$ same as for simple version of log rule.

ex. $\frac{d}{dx} \ln x^5 = \dfrac{\frac{d}{dx}x^5}{x^5} = \dfrac{5\cdot \cancel{x^4}}{\cancel{x^5}\,{}^{1}} = \dfrac{5}{x}$

Problems with log's can often be simplified if you use log properties to simplify before taking the derivative.

Here are a few log properties:

$\ln(1) = 0$, $\ln(e) = 1$,

$$\boxed{\ln x^n = n \cdot \ln x}$$

this one is very useful

Here is the same derivative using log properties before taking the derivative.

$$\frac{d}{dx} \ln x^5 = \frac{d}{dx}\left(5 \ln x\right)$$

$$= 5 \cdot \frac{d}{dx} \ln x$$

$$= 5 \cdot \frac{1}{x} = \frac{5}{x}$$

Odd Topics from Chapter 2.

Δf = Change in $f(x)$

Δ delta means forward difference

this means new − old or future − current value
later − earlier etc.

if x changes from x_1 to x_2 then

$$\Delta x = x_2 - x_1$$

$$\Delta f = f(x_2) - f(x_1)$$

from $\frac{df}{dx} = f'(x)$ multiply both sides by dx and get the differential

$$\boxed{df = f'(x)\,dx}$$ Differential of f(x)

If you replace dx with Δx and df with Δf you get a useful approximation formula.

$$\boxed{\Delta f \approx f'(x) \cdot \Delta x}$$ Differential Approximation

if x changes from x_1 to x_2 then
$\Delta x = x_2 - x_1$
$\Delta f = f(x_2) - f(x_1)$
and you should evaluate $f'(x)$ at x_1 and not x_2

$$\Delta f \approx f'(x_1) \cdot \Delta x$$

or $$\Delta f \approx f'(x_1) \cdot (x_2 - x_1)$$

Odd stuff from chapter 2 Continued:

ex Assume the current Profit for a large company is 80 million dollars and the next day the profit increased by \$800.00. Is this a large increase in profit? Ans. No.

The increase in profit relative to the actual profit is trivial. In this case the Relative Change in profit is what is important.

$$\text{Relative Change} = \frac{\Delta P}{P} = \frac{\$\,\cancel{800}}{\$\,\cancel{80},\cancel{0}00,000}$$

$$= \frac{1}{100{,}000}$$

$$= 1 \cdot 10^{-5} \quad \text{essentially zero.}$$

$$\text{Relative Change} = \frac{\Delta f}{f}$$

It is often useful to put these values in percent form.

ex 10% 10% and 0.1 represent the exact same values

$= 10 \cdot \frac{1}{100}$

$= .1$

Note that $\%$ means times $\frac{1}{100}$

i.e. $\% \equiv \cdot \frac{1}{100}$ percent means per one hundred.

ex. $100\% = 100 \cdot \frac{1}{100} = 1$

so $\boxed{1 = 100\%}$

This gives a useful way to convert any number or formula to percent form, just multiply by 1 (one). It will not change the value of the expression, it will only change the way it looks.

ex. from the earlier example

$$\frac{\Delta P}{P} = \frac{1}{100{,}000} = \frac{1}{100{,}000} \cdot 1$$

$$= \frac{1}{100{,}\cancel{000}} \cdot 1\cancel{00}\%$$

$$= \frac{1}{1{,}000}\% = .001\%$$

So the Relative Change $\frac{\Delta f}{f}$ can be written in percent form. (mult by $1 = 100\%$)

giving

$$\text{Percent Change} = \frac{\Delta f}{f} \cdot 100\%$$

note

Note, in your text book they leave off the percent sign, %, but it is still assumed to be there.

Two more formulas:

$f'(x)$ is Rate of Change in $f(x)$

$$\boxed{\frac{f'(x)}{f} \text{ is Relative Rate of change}}$$

You can multiply this by $1 = 100\%$ to get:

$$\boxed{\frac{f'(x)}{f(x)} \cdot 100\% \text{ is Percentage Rate of change}}$$

Finally, one more thing from Sec 2.5, use $\Delta f \approx f'(x)\cdot\Delta x$ to approximate the above formulas.

Relative Change $\frac{\Delta f}{f}$ can be approximated with

$$\frac{\Delta f}{f} \approx \frac{f'(x)\cdot\Delta x}{f}$$

using the differential approximation to estimate Δf.

Percent Change $\frac{\Delta f}{f}\cdot 100\%$ can be approximated with:

$$\frac{\Delta f}{f}\cdot 100\% \approx \frac{f'(x)\cdot\Delta x}{f}\,100\%$$