Math 165, Fall 2010, Lowman
Week 2 Lec 3

- Chap 1 – More limits
- Chap 2 – definition of derivative

ex. $f(x) = x^2 + x$

find $\lim_{x \to \infty} f(x) = ?$

$x = 10, \quad f(10) = 10^2 + 10 = 100 + 10 = 110$
$x = 100, \quad f(100) = 100^2 + 100 = 10{,}000 + 100 = 10{,}100$
$x = 1000, \quad f(1000) = (1000)^2 + 1000 = 1000000 + 1000$
$= \underbrace{1{,}00\underbrace{1{,}000}_{x}}_{x^2}$

$x = 10^4, \quad f(10^4) = (10^4)^2 + 10^4$
$= 10^8 + 10^4$
$= 100{,}000{,}000 + 10{,}000$
$= \underbrace{100{,}0\underbrace{10{,}000}_{x \text{ term}}}_{x^2 \text{ term}}$

Observe that when x gets large, the higher degree term dominates the limit

$\Rightarrow \lim_{x \to \infty} (x^2 + x) = \lim_{x \to \infty} (x^2) \to (\infty)^2 \to \infty$ undefined

This is a useful property of Polynomials:

$$\lim_{x\to\infty}(\text{Polynomial}) = \lim_{x\to\infty}(\text{highest degree term of Poly})$$

$$\lim_{x\to\infty}\left(a_n \cdot x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0\right) = \lim_{x\to\infty}\left(a_n x^n\right)$$

ex. $$\lim_{x\to\infty}\left[\frac{3x^{10} + \cancel{2x^5} + \cancel{4}}{\cancel{1} + \cancel{2x^4} + \cancel{6x^6} + 9x^{10}}\right]$$ ignore lower degree terms

$$= \lim_{x\to\infty}\left[\frac{3\cancel{x^{10}}}{9\cancel{x^{10}}}\right]$$ intermediate step

$$= \lim_{x\to\infty}\left[\frac{3}{9}\right] = \lim_{x\to\infty}\left[\frac{1}{3}\right] = \boxed{\frac{1}{3}}$$

Here is another method that is often used in textbooks:

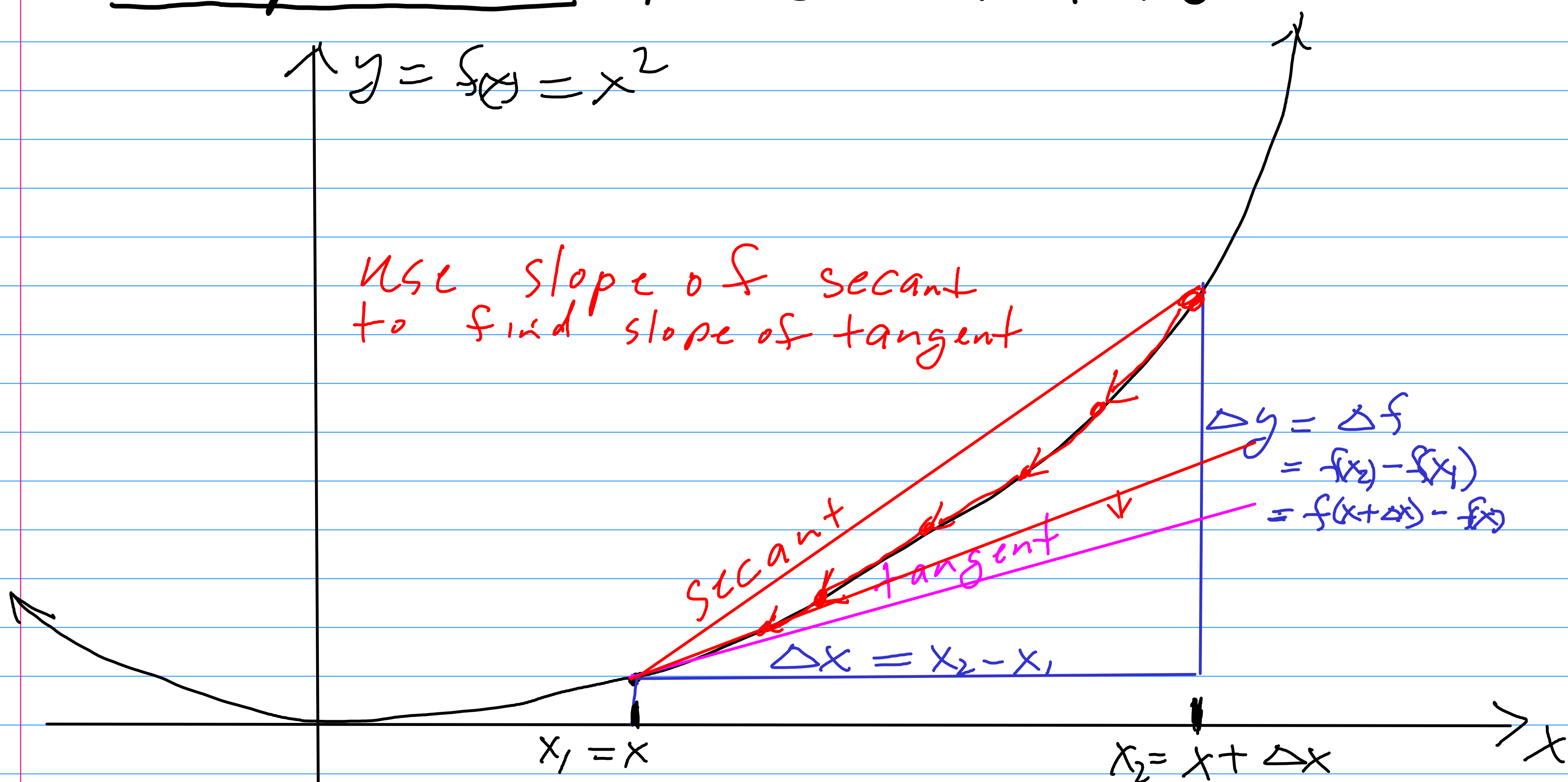
$$\lim_{x\to\infty}\left[\frac{3x^{10} + 2x^5 + 4}{1 + 2x^4 + 6x^6 + 9x^{10}}\right]$$

$$= \lim_{x\to\infty}\left[\frac{\cancel{x^{10}}\left(3 + 2/x^5 + 4/x^{10}\right)}{\cancel{x^{10}}\left(1/x^{10} + 2/x^6 + 6/x^4 + 9\right)}\right]$$

$$= \frac{3 + \cancel{2/(\infty)^5}^{\to 0} + \cancel{4/(\infty)^{10}}^{\to 0}}{\underbrace{\cancel{1/(\infty)^{10}}}_{0} + \underbrace{\cancel{2/(\infty)^6}}_{0} + \underbrace{\cancel{6/(\infty)^4}}_{0} + 9} = 3/9 = \frac{1}{3}$$

Both methods are correct but as a general rule, you should use known properties of polynomials when possible.

Chapter 2: the derivative

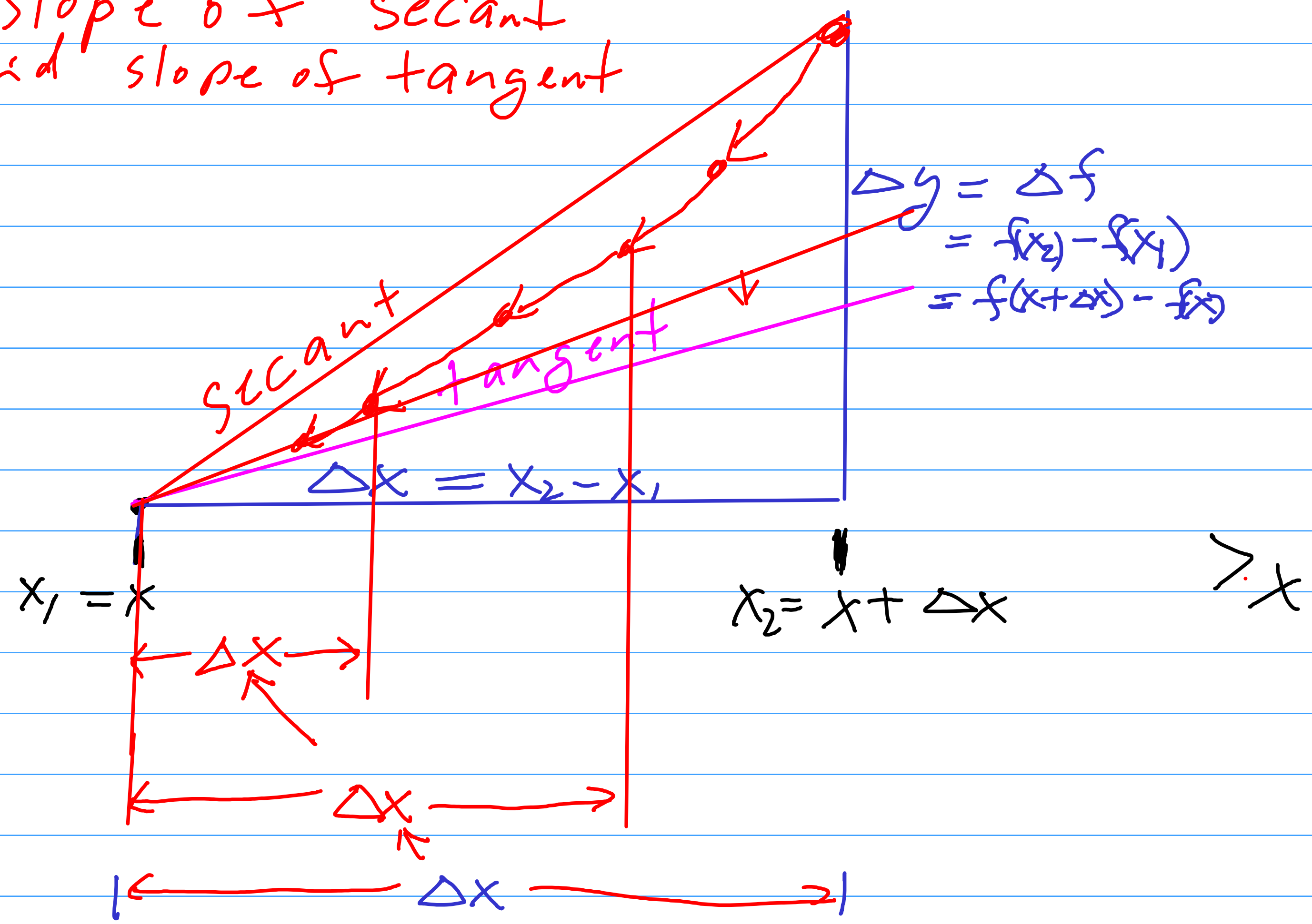


slope of secant $= \dfrac{\Delta y}{\Delta x} = \dfrac{\Delta f}{\Delta x} = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

- this slope is called a <u>difference quotient</u>

Slope of Tangent Line

use slope of secant to find slope of tangent



Observe that as the secant line gets closer to the tangent line Δx is getting close to zero.

$$\text{slope of tangent} = \lim_{\Delta x \to 0} \left[\text{slope of secant line} \right]$$

$$= \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$$

Slope of tangent Line at $x_1 = x$ $\displaystyle = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left[\frac{f(x+\Delta x) - f(x)}{\Delta x} \right]$$

all the same using different notation

- this expression has its own name. It is called the derivative of $f(x)$ at x.
- It is a set of directions for finding the slope of the line tangent to $f(x)$ at x.

Definition of Derivative

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \left[\frac{f(x+\Delta x) - f(x)}{\Delta x} \right]$$

ex. find the slope of $f(x) = mx + b$

This is a good test because we already know the slope of this line is m.

$f(x) = mx + b \qquad f(x+\Delta x) = m(x+\Delta x) + b$

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{[m(x+\Delta x)+b] - [mx+b]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left[\frac{\cancel{mx} + m\cdot\Delta x \cancel{+b} - \cancel{mx} \cancel{-b}}{\Delta x} \right] = \lim_{\Delta x \to 0} \left[m \cdot \frac{\cancel{\Delta x}}{\cancel{\Delta x}} \right]$$

$$= \lim_{\Delta x \to 0} m = m$$ (as expected)