Math 165 L8-W3L2 Friday

Derivative Rules (continued)

Prime notation for derivatives $\frac{df(x)}{dx} = f'(x) = f'$

Here are the derivatives using prime notation:

Assume $K =$ constant $f = f(x)$, $g = g(x)$, $u = u(x)$

$K' = 0$, derivative of a constant is zero

$x' = 1$

$(x^n)' = n \cdot x^{n-1}$, simple power rule

$(K \cdot f)' = K \cdot f'$, Constant times function rule

$(f+g)' = f' + g'$, Sum rule

$(f \cdot g)' = f \cdot g' + g \cdot f'$, product rule

$\left(\frac{f}{g}\right)' = \frac{g \cdot f' - f \cdot g'}{g^2}$, Quotient rule

$(e^x)' = e^x$, Simple exponential rule

$(\ln x)' = \frac{1}{x}$, Simple log rule

General Versions of power, log and exponential rules.

$(u(x)^n)' = n \cdot u(x)^{n-1} \cdot u'(x)$, General Power Rule

$(e^{u(x)})' = e^{u(x)} \cdot u'(x)$ General Exponential Rule

$(\ln u(x))' = \dfrac{u'(x)}{u(x)}$ General Log Rule

Examples

ex1. $(99)' = 0$

ex2. $(3 \cdot x^5)' = 3 \cdot (x^5)' = 3 \cdot 5 \cdot x^4 = 15x^4$

ex3 $(e^x + \ln x)' = (e^x)' + (\ln x)' = e^x + \frac{1}{x}$

ex4. $(e^x \cdot \ln x)'$ - use product rule

$= e^x \cdot (\ln x)' + \ln x \cdot (e^x)'$

$= e^x \cdot \frac{1}{x} + \ln x \cdot e^x$ ans.

$= e^x \left(\frac{1}{x} + \ln x \right)$ simplify ans.

ex5: $\left(\dfrac{2x+3}{3x+4}\right)'$ – use quotient rule

$$= \frac{(3x+4)\cdot \cancel{\boxed{\frac{d}{dx}(2x+3)}} \to 2 - (2x+3)\cancel{\boxed{\frac{d}{dx}(3x+4)}} \to 3}{(3x+4)^2}$$

$$\frac{d}{dx}(2x+3) = \frac{d}{dx}2x + \cancel{\frac{d}{dx}3} \to 0 = 2\cdot \cancel{\frac{dx}{dx}} \to 1 + 0 = 2$$

$$\frac{d}{dx}(3x+4) = \frac{d}{dx}3x + \cancel{\frac{d}{dx}4} \to 0 = 3\cdot \cancel{\frac{dx}{dx}} \to 1 + 0 = 3$$

$$= \frac{(3x+4)\cdot 2 - (2x+3)\cdot 3}{(3x+4)^2}$$

$$= \frac{2\cdot(3x+4) - 3\cdot(2x+3)}{(3x+4)^2}$$

$$= \frac{2\cdot(3x+4) + (-3)\cdot(2x+3)}{(3x+4)^2}$$

$$= \frac{2\cdot 3x + 2\cdot 4 + (-3)\cdot 2x + (-3)(3)}{(3x+4)^2}$$

$$= \frac{\cancel{6x} + \overbrace{8 - \cancel{6x} - 9}^{=-1}}{(3x+4)^2} = \frac{-1}{(3x+4)^2}$$