Math 165, Fall 2010, Lowman
L9 - W4 Monday

- More derivative examples.
- Chain Rule

General Derivative Rules

$$\frac{d}{dx} u(x)^n = n \cdot u(x)^{n-1} \cdot u'(x), \quad \text{Power Rule}$$

$$\frac{d}{dx} e^{u(x)} = e^{u(x)} \cdot u'(x), \quad \text{Exponential Rule}$$

$$\frac{d}{dx} \ln u(x) = \frac{u'(x)}{u(x)}, \quad \text{Log Rule}$$

Note, if $u(x) = x$ then $u'(x) = 1$ and the general rules become the same as the simple versions.

ex. $\frac{d}{dx}(3x^2+x)^5$ - use general power rule

$$= 5 \cdot (3x^2+x)^4 \cdot \boxed{\frac{d}{dx}(3x^2+x)}$$

$$\frac{d}{dx}(3x^2+x) = \frac{d}{dx}3x^2 + \cancel{\frac{dx}{dx}} = 3 \cdot \frac{d}{dx}x^2 + 1$$

$$= 3 \cdot 2x + 1 = 6x + 1$$

$= 5 \cdot (3x^2+x)^4 \cdot (6x+1)$

$= 5(6x+1) \cdot (3x^2+x)^4$

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ex. $\frac{d}{dx} e^{x^2}$ – use general exponential rule

$= e^{x^2} \cdot \frac{d}{dx} x^2$

$= e^{x^2} \cdot 2x$

$= \boxed{2x \cdot e^{x^2}}$

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ex. $\frac{d}{dx} \ln(3x^5+2x)$ –use general log rule

$= \frac{\boxed{\frac{d}{dx}(3x^5+2x)}}{(3x^5+2x)} = \boxed{\frac{15x^4+2}{3x^5+2x}}$ answer

$\frac{d}{dx}(3x^5+2x) = \frac{d}{dx}(3x^5) + \frac{d}{dx}(2x)$

$= 3 \cdot \frac{d}{dx} x^5 + 2 \cdot \cancel{\frac{dx}{dx}}^{\,1}$

$= 3 \cdot 5x^4 + 2$

$= 15x^4 + 2$

Here is a long one.

ex. $\frac{d}{dx}\left[\frac{(2x^2+1)^4}{e^x}\right]$ - first use quotient rule.

$$= \frac{e^x \cdot \boxed{\frac{d}{dx}\left[(2x^2+1)^4\right]} - (2x^2+1)^4 \cdot \frac{d}{dx} e^x}{(e^x)^2}$$

← use general power rule

$\frac{d}{dx} e^x \to e^x$

$(e^x)^2 = e^{2x}$

$$\frac{d}{dx}(2x^2+1)^4 = 4\cdot(2x^2+1)^3 \cdot \frac{d}{dx}(2x^2+1)$$

$$= 4\cdot(2x^2+1)^3 \cdot \left(\frac{d}{dx}2x^2 + \cancel{\frac{d}{dx}1}\right) \quad \frac{d}{dx}1 \to 0$$

$$= 4\cdot(2x^2+1)^3 \cdot \left(2\cdot\frac{d}{dx}x^2 + 0\right)$$

$$= 4\cdot(2x^2+1)^3 \cdot (2\cdot 2x + 0)$$

$$= 4\cdot(2x^2+1)^3 \cdot (4x+0)$$

$$= 4\cdot(2x^2+1)^3 \cdot 4x$$

$$= 4\cdot 4x\cdot(2x^2+1)^3$$

$$= 16x\cdot(2x^2+1)^3$$

$$= \frac{e^x \cdot \left[16x(2x^2+1)^3\right] - (2x^2+1)^4 \cdot e^x}{e^{2x}}$$

(now simplify)

$$= \frac{\cancel{e^x}\left[16x(2x^2+1)^3 - (2x^2+1)^4\right]}{\cancel{e^{2x}} \rightarrow e^x}$$

note: $\frac{e^x}{e^{2x}} = \frac{1}{e^{2x-x}} = \frac{1}{e^x}$

$$= \frac{16x(2x^2+1)^3 - (2x^2+1)^4}{e^x}$$

Now simplify more.

factor out the highest power of $(2x^2+1)$ that is common to both terms in the numerator.

$$= \frac{(2x^2+1)^3\left[16x - (2x^2+1)\right]}{e^x}$$

$$= \frac{(2x^2+1)^3 \cdot \left[-2x^2 + 16x - 1\right]}{e^x}$$

final answer.

Problems that start with the quotient rule but still need the use of the three general rules are good problems for practicing your algebra. If your algebra is weak then do more of these.

Chain Rule

In the definition of the derivative,

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \lim_{\Delta x \to 0} \left[\frac{f(x+\Delta x) - f(x)}{\Delta x}\right]$$

You can think of df as a very small Δf

and

You can think of dx as a very small Δx

So $\frac{df}{dx}$ is just df divided by dx.

Now, if $f(x) = f(u)$, where $u = u(x)$

then you can do the following:

$$\frac{df}{dx} = \frac{df}{1} \cdot \frac{1}{dx}$$

Now multiply by $1 = \frac{du}{du}$

$$= \frac{df}{du} \cdot \frac{du}{dx}$$

This gives a new derivative rule called

$$\boxed{\underline{\text{The Chain Rule:}}\ \frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx}}$$

ex. $f(x) = (3x^4+5x)^7$, $\frac{df}{dx} = ?$

Use chain rule or general power rule.

General Power Rule:

$$\frac{d}{dx}(3x^4+5x)^7 = 7\cdot(3x^4+5x)^6\cdot\frac{d}{dx}(3x^4+5x)$$

$$= 7\cdot(3x^4+5x)^6\cdot(12x^3+5)$$

Chain Rule:

$f(x) = (3x^4+5x)^7$

let $f(x) = f(u)$ where $u = u(x)$

$f(u) = u^7$, $u(x) = 3x^4+5x$

Now use chain rule:

$$\frac{df}{dx} = \frac{df}{du}\cdot\frac{du}{dx}$$

$$= \frac{d}{du}(u^7)\cdot\frac{d}{dx}(3x^4+5x)$$

$\Rightarrow$ use simple rule.

$$= 7u^6 \cdot (12x^3+5)$$

Now replace u with $u = 3x^4+5x$

$$= 7(3x^4+5x)^6 \cdot (12x^3+5)$$

Note, The chain rule was used to derive the general derivative rules from the simple power, exponential and log rules.

It is usually easier to the general power, exponential and log rules when possible.