# Implicit Differentiation <br> Math165: Business Calculus 

Roy M. Lowman

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## Implicit Differentiation: prerequisites

- To understand implicit differentiation it is necessary to understand the chain rule.
- Given: $y=y(x), y$ is a function of $x$
- Simple Power Rule: $\frac{d}{d x} x^{n}=n x^{n-1}$
- General Power Rule: $\frac{d}{d x} \mathbf{y}(\mathbf{x})^{\mathrm{n}}=\mathbf{n} \mathbf{y}^{\mathbf{n - 1}} \cdot \mathrm{y}^{\prime}(\mathrm{x})$, due to chain rule:
- $\frac{d}{d y} y^{n}=n y^{n-1}$ is not the same as $\frac{d}{d x} y^{n}=n y^{n-1} \cdot y^{\prime}(x)$


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- $\ln \frac{d}{d y} y^{n}=n y^{n-1}$ the variable of differentiation is $\mathbf{y}$ (i.e. $\frac{d}{d y}$ )
the same as the variable in $\mathbf{y}^{\mathbf{n}}$ so the simple power rule is used.
(i.e. $\frac{d}{d x}$ ) not the same as the variable in $y^{n}$ so the general power rule is used


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- In $\frac{d}{d x} y^{n}=n y^{n-1} \cdot y^{\prime}(x)$ the variable of differentiation is $x$ (i.e. $\frac{d}{d x}$ ) not the same as the variable in $\mathbf{y}^{\mathbf{n}}$ so the general power rule is used.


## explicit vs implicit function

- $y^{3}=3 x^{4}+x$, gives $y$ as a function of $x$ in implicit form.
- Given a set of $x$ values, you can solve for the corresponding y values, plot the points $(\mathbf{x}, \mathbf{y})$ and construct a graph of the function $\mathbf{y}=\mathbf{y}(\mathrm{x})$.
- The relation gives the function $\mathrm{y}=\mathrm{y}(\mathrm{x})$ but in implicit form.
- In this case, it is possible to explicitly solve for $\mathbf{y}(\mathbf{x})$. This will give the same function $\mathbf{y}=\mathbf{y}(\mathbf{x})$ but in explicit form (the one we usually use).

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y^{3} & =3 x^{4}+x  \tag{1}\\
\left(y^{3}\right)^{\frac{1}{3}} & =\left(3 x^{4}+x\right)^{\frac{1}{3}}  \tag{2}\\
y(x) & =\left(3 x^{4}+x\right)^{\frac{1}{3}} \tag{3}
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## explicit vs implicit differentiation (1)

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1: Solve for $\mathbf{y}(\mathbf{x})$ in explicit form

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Use derivative rules to find $\frac{d y}{d x}$

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2: Use derivative rules to find $\frac{d y}{d x}$

## explicit vs implicit differentiation <br> (2)

2 use derivative rules to find $\frac{d y}{d x}$

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y(x) & =\left(3 x^{4}+x\right)^{\frac{1}{3}}  \tag{7}\\
\frac{d}{d x} y(x) & =\frac{d}{d x}\left(3 x^{4}+x\right)^{\frac{1}{3}}  \tag{8}\\
\frac{d y}{d x} & =\frac{1}{3}\left(3 x^{4}+x\right)^{\left(\frac{1}{3}-1\right)} \frac{d}{d x}\left(3 x^{4}+x\right)  \tag{9}\\
\frac{d y}{d x} & =\frac{1}{3}\left(3 x^{4}+x\right)^{\left(\frac{-2}{3}\right)}\left(12 x^{3}+1\right)  \tag{10}\\
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- end explicit differentiation


## explicit vs implicit differentiation (3)

Problem: given $y^{3}=3 x^{4}+x$, use implicit differentiation to find $\frac{d y}{d x}$

## explicit vs implicit differentiation

 (3)Problem: given $\mathbf{y}^{3}=3 \mathrm{x}^{4}+\mathrm{x}$, use implicit differentiation to find $\frac{d y}{d x}$

1: Take the derivative of both sides w.r.t. x
Use derivative rules, remember $\mathrm{y}=\mathrm{y}(\mathrm{x})$ and the chain rule must be used

The resulting expression will contain terms with $y^{\prime}$. Use algebra to solve for $y^{\prime}$
Implicit differentiation is often used when it is difficult or impossible to solve for $\mathbf{y}=\mathbf{y}(\mathbf{x})$ in explicit form

- $y^{3}=3 x^{4}+x$


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- $y^{3}=3 x^{4}+x$
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- $y^{3}=3 x^{4}+x$
$\frac{d}{d x} y^{3}=\frac{d}{d x}\left(3 x^{4}+x\right)$
$3 y^{2} \cdot y^{\prime}=12 x^{3}+1$, now solve for $y^{\prime}$


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$\mathrm{y}^{\prime}(\mathrm{x})=\frac{12 \mathrm{x}^{3}+1}{3 \mathrm{y}^{2}}$


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$y^{\prime}(x)=\frac{12 x^{3}+1}{3 y^{2}}$
$y^{\prime}(x)=\frac{12 x^{3}+1}{3\left[\left(3 x^{4}+x\right)^{\frac{1}{3}}\right]^{2}}=\frac{12 x^{3}+1}{3\left(3 x^{4}+x\right)^{\frac{2}{3}}}$

Here is the problem from the beginning: $y^{3}=3 x^{4}+x$, use implicit differentiation to find $\frac{d y}{d x}$
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$y^{\prime}(x)=\frac{12 x^{3}+1}{3 y^{2}}$
This gives $y^{\prime}(x)$ as a function of both $x$ and $y$. In this case it is easy to solve for $\mathbf{y}(\mathbf{x})$ using the original function in implicit form. Using the expression found earlier for $\mathbf{y}$ gives:


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$\frac{3 y^{2} \cdot y^{\prime}}{3 y^{2}}=\frac{12 x^{3}+1}{3 y^{2}}$
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This gives $y^{\prime}(x)$ as a function of both $x$ and $y$. In this case it is easy to solve for $\mathbf{y}(\mathbf{x})$ using the original function in implicit form.
Using the expression found earlier for $y$ gives:


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$y^{\prime}(x)=\frac{12 x^{3}+1}{3\left[\left(3 x^{4}+x\right)^{\frac{1}{3}}\right]^{2}}$

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$3 y^{2} \cdot y^{\prime}=12 x^{3}+1$, now solve for $y^{\prime}$
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$y^{\prime}(x)=\frac{12 x^{3}+1}{3 y^{2}}$
This gives $\mathbf{y}^{\prime}(\mathbf{x})$ as a function of both $\mathbf{x}$ and $\mathbf{y}$. In this case it is easy to solve for $\mathbf{y}(\mathbf{x})$ using the original function in implicit form. Using the expression found earlier for $y$ gives:
$y^{\prime}(x)=\frac{12 x^{3}+1}{3\left[\left(3 x^{4}+x\right)^{\frac{1}{3}}\right]^{2}}$
$y^{\prime}(x)=\frac{12 x^{3}+1}{3\left(3 x^{4}+x\right)^{\frac{1}{3}}}$

## explicit vs implicit differentiation

Here is the problem from the beginning: $y^{3}=3 x^{4}+x$, use implicit differentiation to find $\frac{d y}{d x}$
$y^{3}=3 x^{4}+x$
$\frac{d}{d x} y^{3}=\frac{d}{d x}\left(3 x^{4}+x\right)$
$3 y^{2} \cdot y^{\prime}=12 x^{3}+1$, now solve for $y^{\prime}$
$\frac{3 y^{2} \cdot y^{\prime}}{3 y^{2}}=\frac{12 x^{3}+1}{3 y^{2}}$
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## Implicit Differentiation

## typical exam problem

Problem: Find the slope of the line tangent to the graph of $3 x y^{2}+4 y=10$ at the point $(x, y)=(2,1)$

```
Solution: use implicite differentiation
3xy2}+4y=1
d
\frac{d}{dx}}3x\mp@subsup{y}{}{2}+\frac{d}{dx}4y=
dx
3x}\cdot\frac{\textrm{d}}{\textrm{dx}}(\mp@subsup{y}{}{2})+\mp@subsup{\textrm{y}}{}{2}\cdot\frac{\textrm{d}}{\textrm{dx}}(3x)+4\mp@subsup{y}{}{\prime}=
3x}\cdot(2\mp@subsup{y}{}{2}\mp@subsup{y}{}{\prime})+\mp@subsup{y}{}{2}\cdot3+4\mp@subsup{y}{}{\prime}=
6x\mp@subsup{y}{}{2}\mp@subsup{y}{}{\prime}+3\mp@subsup{y}{}{2}+4\mp@subsup{y}{}{\prime}=0, subtract 3y2 from both sides
6x\mp@subsup{y}{}{2}\cdot\mp@subsup{y}{}{\prime}+4\mp@subsup{y}{}{\prime}=-3\mp@subsup{y}{}{2},\mathrm{ factor out }\mp@subsup{y}{}{\prime}\mathrm{ on the left}
y'}\cdot(6x\mp@subsup{y}{}{2}+4)=-3\mp@subsup{y}{}{2}\mathrm{ , divide both sides by }6x\mp@subsup{y}{}{2}+
\mp@subsup{y}{}{\prime}}=\frac{-3\mp@subsup{y}{}{2}}{6x\mp@subsup{y}{}{2}+4},\mathrm{ evaluate the slope at point (x,y)=(2,1)
y'}=\frac{-3(1\mp@subsup{)}{}{2}}{6(2)(1\mp@subsup{)}{}{2}+4}=-\frac{3}{16
```


## Implicit Differentiation

## typical exam problem

Problem: Find the slope of the line tangent to the graph of $3 x y^{2}+4 y=10$ at the point $(x, y)=(2,1)$
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Solution: use implicite differentiation:

$$
\begin{aligned}
& 3 x y^{2}+4 y=10 \\
& \frac{d}{d x}\left(3 x y^{2}+4 y\right)=\frac{d}{d x}(10)
\end{aligned}
$$



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& \frac{d}{d x} 3 x y^{2}+\frac{d}{d x} 4 y=0
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& \frac{d}{d x}\left(3 x \cdot y^{2}\right)+4 y^{\prime}=0
\end{aligned}
$$



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$$

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$$

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\frac{d}{d x} 3 x y^{2}+\frac{d}{d x} 4 y=0
$$

$$
\frac{d x}{d x}\left(3 x \cdot y^{2}\right)+4 y^{\prime}=0
$$

$$
3 x \cdot \frac{d}{d x}\left(y^{2}\right)+y^{2} \cdot \frac{d}{d x}(3 x)+4 y^{\prime}=0
$$

$$
6 x y^{2} y^{\prime}+3 y^{2}+4 y^{\prime}=0, \text { subtract } 3 y^{2} \text { from both sides }
$$

$$
6 x y^{2} \cdot y^{\prime}+4 y^{\prime}=-3 y^{2}, \text { factor out } y^{\prime} \text { on the left }
$$

$$
\begin{aligned}
& y^{\prime} \cdot\left(6 x y^{2}+4\right)=-3 y^{2}, \text { divide both sides by } 6 x y^{2}+4 \\
& y^{\prime}=\frac{-3 y^{2}}{6 x y^{2}+4}, \text { evaluate the slope at point }(x, y)=(2,1)
\end{aligned}
$$

$$
y^{\prime}=\frac{-3(1)^{2}}{6(2)(1)^{2}+4}=-\frac{3}{16}
$$

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& 3 x \cdot \frac{d}{d x}\left(y^{2}\right)+y^{2} \cdot \frac{d}{d x}(3 x)+4 y^{\prime}=0 \\
& 3 x \cdot\left(2 y^{2} y^{\prime}\right)+y^{2} \cdot 3+4 y^{\prime}=0
\end{aligned}
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6 x y^{2} y^{\prime}+3 y^{2}+4 y^{\prime}=0, \text { subtract } 3 y^{2} \text { from both sides }
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& y^{\prime} \cdot\left(6 x y^{2}+4\right)=-3 y^{2}, \text { divide both sides by } 6 x y^{2}+4 \\
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\frac{d}{d x}\left(3 x y^{2}+4 y\right)=\frac{d}{d x}(10)
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$$
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3 x \cdot\left(2 y^{2} y^{\prime}\right)+y^{2} \cdot 3+4 y^{\prime}=0
$$

$$
\mathbf{6 x y ^ { 2 }} \mathbf{y}^{\prime}+\mathbf{3} \mathbf{y}^{\mathbf{2}}+\mathbf{4} \mathbf{y}^{\prime}=\mathbf{0}, \text { subtract } 3 y^{2} \text { from both sides }
$$



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$3 x \cdot\left(2 y^{2} y^{\prime}\right)+y^{2} \cdot 3+4 y^{\prime}=0$
$\mathbf{6 x} \mathbf{y}^{2} \mathbf{y}^{\prime}+3 \mathbf{y}^{2}+\mathbf{4} \mathbf{y}^{\prime}=\mathbf{0}$, subtract $\mathbf{3} \mathbf{y}^{\mathbf{2}}$ from both sides


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$6 x y^{2} y^{\prime}+3 y^{2}+4 y^{\prime}=0$, subtract $3 y^{2}$ from both sides
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$y^{\prime} \cdot\left(6 x y^{2}+4\right)=-3 y^{2}$,


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$y^{\prime} \cdot\left(6 x y^{2}+4\right)=-3 y^{2}$, divide both sides by $\mathbf{6 x y ^ { 2 }}+\mathbf{4}$


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$y^{\prime}=\frac{-3(1)^{2}}{6(2)(1)^{2}+4}=-\frac{3}{16}$

