Implicit Differentiation

Math165: Business Calculus

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Implicit Differentiation: prerequisites

- To understand **implicit differentiation** it is necessary to understand the chain rule.
- Given: $y = y(x)$, y is a function of x
- Simple Power Rule: $\frac{d}{dx}x^n = nx^{n-1}$
- General Power Rule: $\frac{d}{dx}y(x)^n = ny^{n-1} \cdot y'(x)$, due to chain rule:
  - $\frac{d}{dx}y^n = \frac{d}{dy}y^n \cdot \frac{dy}{dx} = ny^{n-1} \cdot y'(x)$
- $\frac{d}{dy}y^n = ny^{n-1}$ is **not** the same as $\frac{d}{dx}y^n = ny^{n-1} \cdot y'(x)$
  - In $\frac{d}{dy}y^n = ny^{n-1}$ the variable of differentiation is y (i.e. $\frac{d}{dy}$) the same as the variable in y^n so the simple power rule is used.
  - In $\frac{d}{dx}y^n = ny^{n-1} \cdot y'(x)$ the variable of differentiation is x (i.e. $\frac{d}{dx}$) **not** the same as the variable in y^n so the general power rule is used.

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- $y^3 = 3x^4 + x$, gives y as a function of x in implicit form.
- Given a set of x values, you can solve for the corresponding y values , plot the points (x, y) and construct a graph of the function $y = y(x)$.
- The relation gives the function $y = y(x)$ but in *implicit* form.
- In this case, it is possible to explicitly solve for $y(x)$. This will give the same function $y = y(x)$ but in *explicit* form (the one we usually use).

$$y^3 = 3x^4 + x \quad (1)$$

$$(y^3)^{\frac{1}{3}} = (3x^4 + x)^{\frac{1}{3}} \quad (2)$$

$$y(x) = (3x^4 + x)^{\frac{1}{3}} \quad (3)$$

explicit vs implicit function

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explicit vs implicit differentiation (1)

Problem: given $\mathbf{y^3 = 3x^4 + x}$,
use explicit differentiation to find $\mathbf{\frac{dy}{dx}}$

1: Solve for y(x) in explicit form

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- $$y(x) = (3x^4 + x)^{\frac{1}{3}} \quad (7)$$
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  $$\frac{dy}{dx} = \frac{12x^3 + 1}{3(3x^4 + x)^{(\frac{2}{3})}} \quad (12)$$
- end explicit differentiation

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Problem: given $\mathbf{y^3 = 3x^4 + x}$,
use **implicit differentiation** to find $\frac{\mathbf{dy}}{\mathbf{dx}}$

1: Take the derivative of both sides w.r.t. x

2: Use derivative rules, remember $y = y(x)$ and the chain rule must be used.

3: The resulting expression will contain terms with y'. Use algebra to solve for y'

note: Implicit differentiation is often used when it is difficult or impossible to solve for $y = y(x)$ in explicit form.

- $y^3 = 3x^4 + x$
  $\frac{d}{dx} y^3 = \frac{d}{dx}(3x^4 + x)$
  $3y^2 \cdot y' = 12x^3 + 1$, now solve for y'
  $y'(x) = \frac{12x^3+1}{3y^2}$
  $y'(x) = \frac{12x^3+1}{3[(3x^4+x)^{\frac{1}{3}}]^2} = \frac{12x^3+1}{3(3x^4+x)^{\frac{2}{3}}}$

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explicit vs implicit differentiation (3)

Problem: given $\mathbf{y^3 = 3x^4 + x}$,
use **implicit differentiation** to find $\frac{dy}{dx}$

1: Take the derivative of both sides w.r.t. $\mathbf{x}$

2: Use derivative rules, remember $\mathbf{y = y(x)}$ and the chain rule must be used.

3: The resulting expression will contain terms with $\mathbf{y'}$. Use algebra to solve for $\mathbf{y'}$

note: Implicit differentiation is often used when it is difficult or impossible to solve for $\mathbf{y = y(x)}$ in explicit form.

- $y^3 = 3x^4 + x$
  $\frac{d}{dx}y^3 = \frac{d}{dx}(3x^4 + x)$
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Here is the problem from the beginning: $y^3 = 3x^4 + x$,
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$\frac{3y^2 \cdot y'}{3y^2} = \frac{12x^3+1}{3y^2}$

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This gives $y'(x)$ as a function of both x and y. In this case it is easy to solve for $y(x)$ using the original function in implicit form. Using the expression found earlier for y gives:

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Implicit Differentiation

typical exam problem

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$y' = \frac{-3y^2}{6xy^2+4}$, evaluate the slope at point $(x, y) = (2, 1)$

$y' = \frac{-3(1)^2}{6(2)(1)^2+4} = -\frac{3}{16}$

Problem: Find the slope of the line tangent to the graph of $3xy^2 + 4y = 10$ at the point $(x, y) = (2, 1)$

Solution: use implicite differentiation:

$3xy^2 + 4y = 10$

$\frac{d}{dx}(3xy^2 + 4y) = \frac{d}{dx}(10)$

$\frac{d}{dx}3xy^2 + \frac{d}{dx}4y = 0$

$\frac{d}{dx}(3x \cdot y^2) + 4y' = 0$

$3x \cdot \frac{d}{dx}(y^2) + y^2 \cdot \frac{d}{dx}(3x) + 4y' = 0$

$3x \cdot (2y^2y') + y^2 \cdot 3 + 4y' = 0$

$6xy^2y' + 3y^2 + 4y' = 0$, subtract $3y^2$ from both sides

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