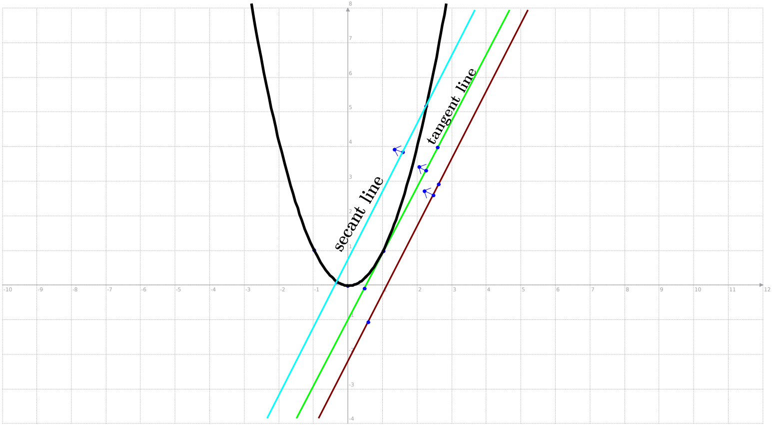
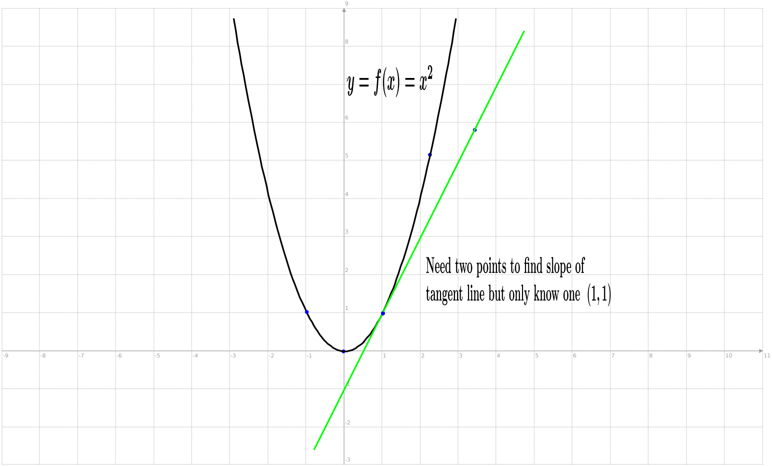
Slope of Tangent

secant line



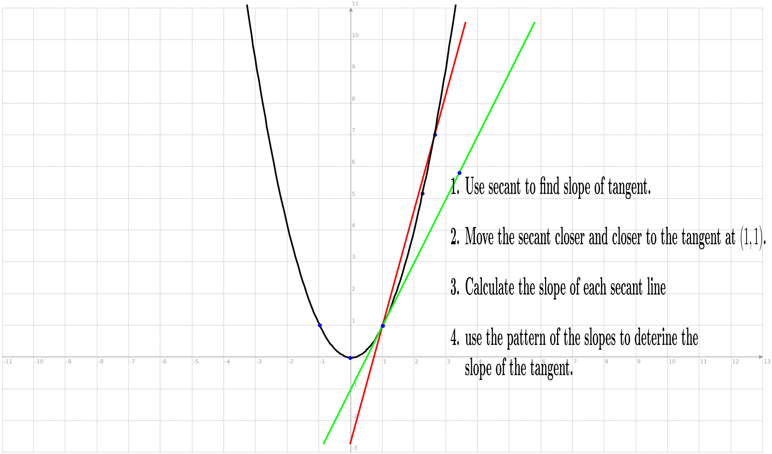
Slope of Tangent

secant line



Slope of Tangent

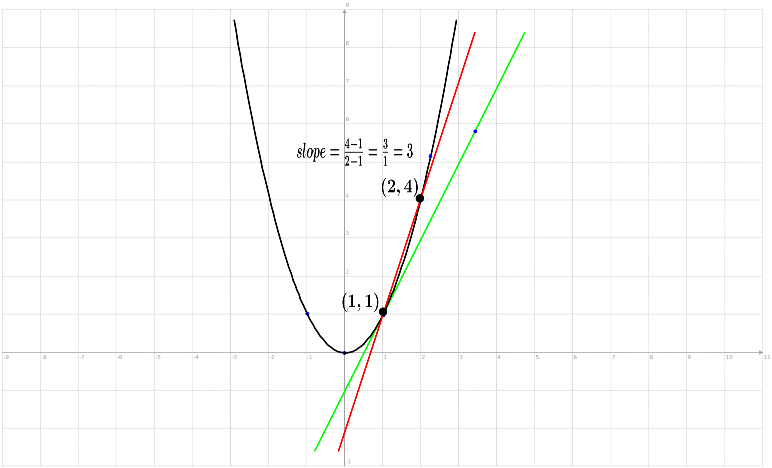
secant line



1. Use secant to find slope of tangent.
2. Move the secant closer and closer to the tangent at $(1,1)$.
3. Calculate the slope of each secant line
4. use the pattern of the slopes to deterine the slope of the tangent.

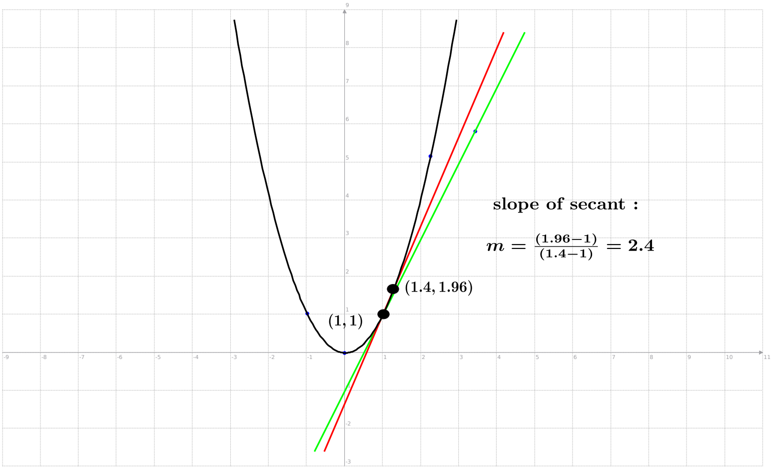
Slope of Tangent

secant line



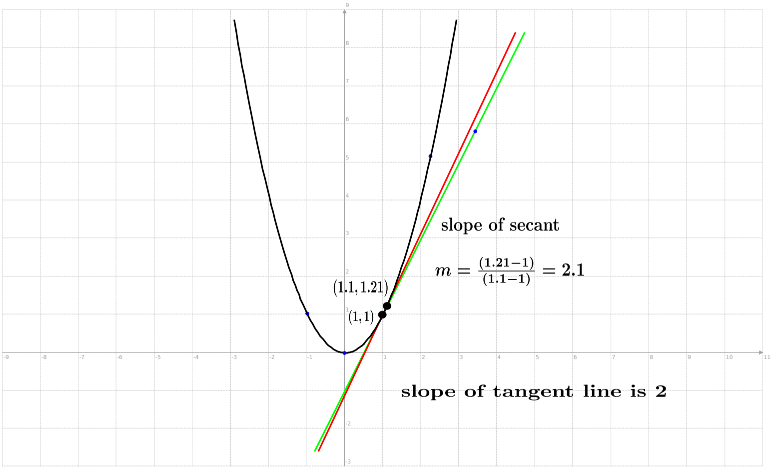
Slope of Tangent

secant line



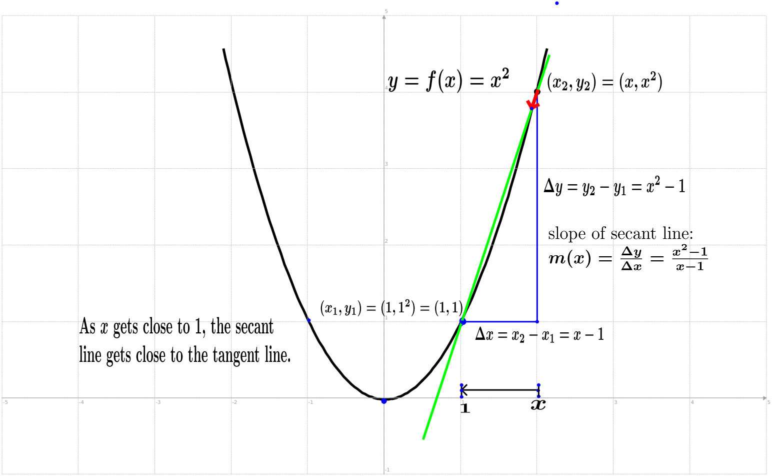
Slope of Tangent

secant line



Slope of Tangent

secant line



Slope of Tangent

Right Hand Limit

$\mathbf{x}$	$\mathbf{2}$	1.5	1.1	1.01	1.001	1.0001	$\longrightarrow \mathbf{1}^+ \longrightarrow \mathbf{1}$
$\mathbf{m = \frac{x^2-1}{x-1}}$	3	2.5	2.2	2.01	2.001	2.0001	$\longrightarrow \mathbf{2}^+ \longrightarrow \mathbf{2}$

- The right hand limit (RHL) of $\mathbf{m(x) = \frac{x^2-1}{x-1}}$ as $\mathbf{x}$ approaches $\mathbf{1}$ from the right is $\mathbf{2}$.

Slope of Tangent

Right Hand Limit

$\mathbf{x}$	$\mathbf{2}$	1.5	1.1	1.01	1.001	1.0001	$\longrightarrow \mathbf{1}^{+} \longrightarrow \mathbf{1}$
$\mathbf{m = \frac{x^2-1}{x-1}}$	3	2.5	2.2	2.01	2.001	2.0001	$\longrightarrow \mathbf{2}^{+} \longrightarrow \mathbf{2}$

- The right hand limit (RHL) of $\mathbf{m(x) = \frac{x^2-1}{x-1}}$ as $\mathbf{x}$ approaches $\mathbf{1}$ from the right is $\mathbf{2}$.
- written

$$\lim_{x \to 1^{+}} \frac{x^2 - 1}{x - 1} = 2$$

Slope of Tangent

Right Hand Limit

x	2	1.5	1.1	1.01	1.001	1.0001	$\longrightarrow 1^+ \longrightarrow 1$
$m = \frac{x^2-1}{x-1}$	3	2.5	2.2	2.01	2.001	2.0001	$\longrightarrow 2^+ \longrightarrow 2$

- The right hand limit (RHL) of $m(x) = \frac{x^2-1}{x-1}$ as x approaches 1 from the right is 2.
- written

$$\lim_{x \longrightarrow 1^+} \frac{x^2-1}{x-1} = 2$$

- It seems that the slope of the line tangent to $f(x) = x^2$ at $x = 1$ is 2

Slope of Tangent

Right Hand Limit

x	2	1.5	1.1	1.01	1.001	1.0001	$\to 1^+ \to 1$
$m = \frac{x^2-1}{x-1}$	3	2.5	2.2	2.01	2.001	2.0001	$\to 2^+ \to 2$

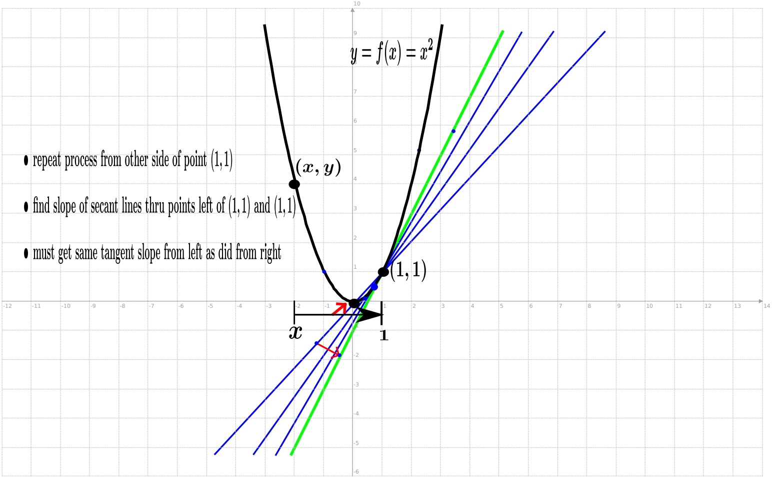
- The right hand limit (RHL) of $m(x) = \frac{x^2-1}{x-1}$ as x approaches 1 from the right is 2.
- written

$$\lim_{x \to 1^+} \frac{x^2-1}{x-1} = 2$$

- It seems that the slope of the line tangent to $f(x) = x^2$ at $x = 1$ is 2
- It is often necessary to check if get the same result when x approaches 1 from the left.

Slope of Tangent

secant line



Slope of Tangent

Left Hand Limit

$\mathbf{x}$	$\mathbf{.9}$	.99	.999	.9999	$\rightarrow \mathbf{1^-} \rightarrow \mathbf{1}$
$\mathbf{m = \frac{x^2-1}{x-1}}$	1.9	1.99	1.999	1.9999	$\rightarrow \mathbf{2^-} \rightarrow \mathbf{2}$

- The left hand limit (LHL) of $\mathbf{m(x) = \frac{x^2-1}{x-1}}$ as $\mathbf{x}$ approaches $\mathbf{1}$ from the left is $\mathbf{2}$.

Slope of Tangent

Left Hand Limit

$\mathbf{x}$	$\mathbf{.9}$	.99	.999	.9999	$\to \mathbf{1^-} \to \mathbf{1}$
$\mathbf{m = \frac{x^2-1}{x-1}}$	1.9	1.99	1.999	1.9999	$\to \mathbf{2^-} \to \mathbf{2}$

- The left hand limit (LHL) of $\mathbf{m(x) = \frac{x^2-1}{x-1}}$ as $\mathbf{x}$ approaches $\mathbf{1}$ from the left is $\mathbf{2}$.
- Written

$$\lim_{x \to 1^-} \frac{x^2 - 1}{x - 1} = 2$$

Slope of Tangent
Left Hand Limit

$\mathbf{x}$	$.9$	$.99$	$.999$	$.9999$	$\rightarrow 1^- \rightarrow 1$
$\mathbf{m = \frac{x^2-1}{x-1}}$	1.9	1.99	1.999	1.9999	$\rightarrow 2^- \rightarrow 2$

- The left hand limit (LHL) of $\mathbf{m(x) = \frac{x^2-1}{x-1}}$ as $\mathbf{x}$ approaches $\mathbf{1}$ from the left is $\mathbf{2}$.
- Written
$$\lim_{x \to 1^-} \frac{x^2 - 1}{x - 1} = 2$$
- Again it seems that the slope of the line tangent to $\mathbf{f(x) = x^2}$ at $\mathbf{x = 1}$ is $\mathbf{2}$

Slope of Tangent

Existance of Limits

- The right hand limit exists and is **2**

$$\lim_{x \longrightarrow 1^{+}} \frac{x^2 - 1}{x - 1} = 2$$

Slope of Tangent

Existance of Limits

- The right hand limit exists and is **2**

$$\lim_{x \to 1^+} \frac{x^2 - 1}{x - 1} = 2$$

- The left hand limit exists and is **2**

$$\lim_{x \to 1^-} \frac{x^2 - 1}{x - 1} = 2$$

Slope of Tangent

Existance of Limits

- The right hand limit exists and is **2**

$$\lim_{x \to 1^+} \frac{x^2 - 1}{x - 1} = 2$$

- The left hand limit exists and is **2**

$$\lim_{x \to 1^-} \frac{x^2 - 1}{x - 1} = 2$$

- The LHL and RHL exist and are both equal to **2**.

Slope of Tangent

Existance of Limits

- The right hand limit exists and is **2**

$$\lim_{x \to 1^+} \frac{x^2 - 1}{x - 1} = 2$$

- The left hand limit exists and is **2**

$$\lim_{x \to 1^-} \frac{x^2 - 1}{x - 1} = 2$$

- The LHL and RHL exist and are both equal to **2**.
- Therefore the two sided limit exists and is **2**

Slope of Tangent

Existance of Limits

- The right hand limit exists and is **2**

$$\lim_{x\to 1^+} \frac{x^2-1}{x-1} = 2$$

- The left hand limit exists and is **2**

$$\lim_{x\to 1^-} \frac{x^2-1}{x-1} = 2$$

- The LHL and RHL exist and are both equal to **2**.
- Therefore the two sided limit exists and is **2**
- Written

$$\lim_{x\to 1} \frac{x^2-1}{x-1} = 2$$