Second Derivative

Math165: Business Calculus

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Spring 2010, Week6 Lec2

Definition (first derivative)

$$\frac{df}{dx} = f' = \lim_{\Delta x -> 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \quad (1)$$

$$= \lim_{\Delta x -> 0} \frac{\Delta f}{\Delta x} \quad (2)$$

$$\approx \frac{\Delta f}{\Delta x} \text{ average slope of } \mathbf{f(x)} \text{ over delta x, } \mathbf{f'_{avg}} \quad (3)$$

$$(4)$$

- $f'(x)$ is the slope of the function $f(x)$
- $f'(x)$ is the rate of change of the function $f(x)$ w.r.t x
- $f'(x)$ is $(+)$ where $f(x)$ is increasing.
- $f'(x)$ is $(-)$ where $f(x)$ is decreasing.
- $f'(x)$ can be used to find intervals where $f(x)$ is increasing/decreasing
- $f'(x) = 0$ **or undefined** where the sign of the slope can change, i.e. at CNs
- $f'(x) = 0$ at critical points
- The **First Derivative Test** can be used to determine what kind of critical points.
- $f'(x)$ can tell you a lot about a function $f(x)$
- $f'_{avg} = \frac{\Delta f}{\Delta x}$ can be used to extimate $f'(x)$

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Definition (second derivative)

$$\frac{d^2f}{dx^2} = \frac{d}{dx}f'(x) = \lim_{\Delta x -> 0} \frac{f'(x + \Delta x) - f(x)'}{\Delta x} \quad (5)$$

$$= \lim_{\Delta x -> 0} \frac{\Delta f'}{\Delta x} \quad (6)$$

$$\approx \frac{\Delta f'}{\Delta x}\text{average slope of } f'\text{over}\Delta x, \quad f''_{avg} \quad (7)$$

$$(8)$$

- $f''(x)$ gives the **concavity** of function $f(x)$
- $f''(x)$ is the rate of change of slope w.r.t x
- $f''(x)$ is $(+)$ where $f(x)$ is concave up. (holds H_2O)
- $f''(x)$ is $(-)$ where $f(x)$ is concave down. (makes letter **A**)
- $f''(x)$ can be used to find intervals where $f(x)$ is **concave up** or **concave down**
- $f''(x) = 0$ **or undefined** where the concavity can change
- $f''(x) = 0$ at **inflection points**, but must check if actually IP.
- The **Second Derivative Test** can be used to determine what type of critical points where $f'(x) = 0$.
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TBA A few slides with notes from the lecture are missing here and will be added latter. The missing slides show how f" is related to the curvature of a graph, inflection points and how the second derivative test works.

Typical Exam Problem:
Given $\mathbf{f(x) = x^4 - 6x^2 - 12}$, use $\mathbf{f''}$ to determine where the graph of $\mathbf{f(x)}$ is concave up and where it is concave down.

1st set $\mathbf{f'' = 0}$ to determine where the concavity can change.

2nd evaluate $\mathbf{f''(x)}$ at one test point in each interval.

- If $f'' = (+)$ (holds water) at one test point in an interval then $f(x)$ is concave up at that test point and at every point in the same interval.
- If $f'' = (-)$ (makes letter A) at one test point in an interval then $f(x)$ is concave down at that test point and at every point in the same interval.
- Repeat for one test point in each interval. Organize your work by drawing a number line with boundaries etc.

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- $\mathbf{f' = 4x^3 - 12x}$
- $\mathbf{f'' = 12x^2 - 12 = 12(x^2 - 1) = 12(x-1)(x+1)}$
- solve $\mathbf{f'' = 12(x-1)(x+1) = 0}$ gives boundaries where concavity can change at $\mathbf{x = 1}$, and $\mathbf{x = -1}$

2nd evaluate $\mathbf{f''(x)}$ at one test point in each interval.

- Convenient test points $\mathbf{x = -2, 0}$ and $\mathbf{2}$
- $\mathbf{f''(-2) = 12((-2)-1)((-2)+1) = (-)(-) = (+)}$ Positive (holds water) $\Rightarrow$ concave up in this interval.
- $\mathbf{f''(0) = 12((0)-1)((0)+1) = (-)(+) = (-)}$ Negative (makes letter A) $\Rightarrow$ concave down in this interval.
- $\mathbf{f''(2) = 12((2)-1)((2)+1) = (+)(+) = (+)}$ Positive (holds water) $\Rightarrow$ concave up in this interval.
- now organize results by drawing a number line with boundaries etc.

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- now organize results by drawing a number line with boundaries etc.

second derivative

example: find intervals concave up/down

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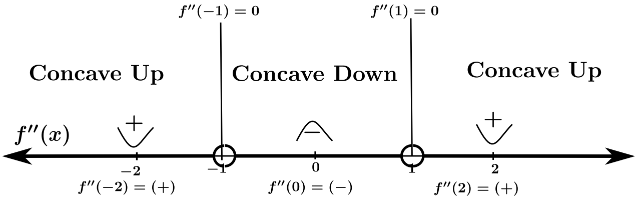

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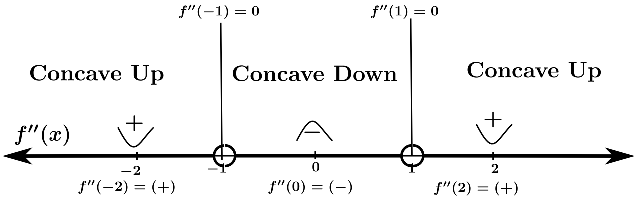

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- The point on the graph $(\mathbf{x_c, f(x_c)})$ is a critical point (**CP**).
- The second derivative can be used to determine what kind of **CP**.

Definition (Second Derivative Test)

- Given $\mathbf{x_c}$ is a critical number where $\mathbf{f'(x_c) = 0}$ then if:
- $f''(x_c) = (+)$ (holds water) then the **CP** is a **Relative Minimum**
- $f''(x_c) = (-)$ (makes letter A) then the **CP** is a **Relative Maximum**
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Typical Exam Problem:
Given $\mathbf{f(x) = 100 - (x - 4)^2}$, find all critical numbers $\mathbf{x_c}$, find all critical points, then use the Second Derivative Test to determine what kind of CP

- $f' = -2(x - 4) = 2(4 - x) = 0$ gives $x_c = 4$
- Critical Point is $(4, 100)$
- $f''(x) = -2$
- now use the second derivative test
- $f''(4) = -2 = (-)$ Negative (makes letter A) indicates that the $\mathbf{CP}$ $(4, 100)$ is a relative maximum.

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- An inflection point will occur where $\mathbf{f''(x) = 0}$.
- However, the converse may not be true so you must check the sign of $\mathbf{f''(x)}$ on each side of the point where $\mathbf{f''(x) = 0}$.
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second derivative
example: inflection point

Typical exam problem:
Given $\mathbf{f(x) = (x-2)^3 + 1}$, find all inflection points (if any).

- $f' = 3(x-2)^2$
- $f'' = 6(x-2)$
- solve $f'' = 6(x-2) = 0$ for possible IPs.
- There may be an inflection at $x = 2$
- The point $(2, 1)$ may be an IP, need to check the sign of $f''(x)$ on both sides of the point.
- $f''(1) = 6(1-2) = (-)$ Negative (letter A) $\Rightarrow$ concave down on left of point
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