MA 430, THIRD HOMEWORK SET, DUE WEDNESDAY, SEPTEMBER 23RD.

1. Exercise

For a subset $A \subseteq \mathbb{N}$, we let $[A]^2 = \{(n,m) \in \mathbb{N}^2 \mid n,m \in A \ \& \ n < m\}$. We can think of $[A]^2$ as being the set of 2-element subsets of A. Recall the infinite version of Ramsey's theorem:

Theorem 1. *Suppose $c\colon [\mathbb{N}]^2 \to \{0,1\}$ is a colouring. Then there is an infinite subset $A \subseteq \mathbb{N}$ such that $[A]^2$ is* monochromatic, *i.e., such that $c|_{[A]^2}$ is constant.*

Use the Compactness Theorem for propositional logic to show the following finite version of Ramsey's Theorem:

Theorem 2. *For any $k \in \mathbb{N}$ there is an $n \in \mathbb{N}$ such that if*

$$c\colon [\{1,2,\dots,n\}]^2 \to \{0,1\}$$

is a colouring, there is a k-element subset $A \subseteq \{1,2,\dots,n\}$ such that $[A]^2$ is monochromatic.

2. Exercise

Suppose L is a propositional language and $\mathcal{A}$ and $\mathcal{B}$ are sets of L-formulas. We write

$$\models \bigwedge \mathcal{A} \leftrightarrow \bigvee \mathcal{B}$$

if whenever v is a valuation of L satisfying $\mathcal{A}$, then there is some $B \in \mathcal{B}$ such that $v(B) = T$ and whenever v is a valuation satisfying some $B \in \mathcal{B}$ then v satisfies $\mathcal{A}$.

Show that if

$$\models \bigwedge \mathcal{A} \leftrightarrow \bigvee \mathcal{B}$$

then there are finite subsets $\mathcal{A}_0 \subseteq \mathcal{A}$ and $\mathcal{B}_0 \subseteq \mathcal{B}$ such that

$$\models \bigwedge \mathcal{A}_0 \leftrightarrow \bigvee \mathcal{B}_0.$$

Hint: Consider the following set of formulas

$$\mathcal{C} = \{A, \neg B \mid A \in \mathcal{A}, B \in \mathcal{B}\}.$$