# MA 430, FOURTH HOMEWORK SET, DUE WEDNESDAY, SEPTEMBER 30TH.

## 1. Exercise

Let $A$ be the formula $((P \wedge Q) \rightarrow (R \leftrightarrow (\neg P \vee Q)))$.

(a) Find a tautologically equivalent formula containing only the connectives $\rightarrow$ and $\leftrightarrow$.

(b) Find a formula in disjunctive normal form that is tautologically equivalent to $A$.

(c) Find a formula in conjunctive normal form that is tautologically equivalent to $A$.

## 2. Exercise

(a) Show that no matter how parentheses are distributed in

$$P_1 \leftrightarrow P_2 \leftrightarrow P_3 \leftrightarrow \ldots \leftrightarrow P_{2n},$$

the resulting formula is true if and only if an *even* number of the $P_i$ are true.

(b) Show that no matter how parentheses are distributed in

$$P_1 \leftrightarrow P_2 \leftrightarrow P_3 \leftrightarrow \ldots \leftrightarrow P_{2n+1},$$

the resulting formula is true if and only if an *odd* number of the $P_i$ are true.

Conclude that any distribution of parentheses in

$$P_1 \leftrightarrow P_2 \leftrightarrow P_3 \leftrightarrow \ldots \leftrightarrow P_n$$

lead to logically equivalent formulas.

## 3. Exercise

Let $P$ and $Q$ be distinct propositional variables and for every two-place logical connective $x$, let

$$A_x = (P\ x\ (Q\ x\ P))$$

and

$$B_x = ((P\ x\ Q)\ x\ \neg(P\ x\ Q)).$$

A formula is said to be an *antilogy* if its negation is a tautology.

- Decide which of the above formulas are tautologies or antilogies when

$$x = \vee \qquad x = \wedge \qquad x = \leftrightarrow$$

$$x = \rightarrow \qquad x = \uparrow \qquad x = \downarrow$$