MA 430, SIXTH HOMEWORK SET, DUE WEDNESDAY, OCTOBER 21ST.

1. Exercise

In the US there are two parties; democrats and republicans. All of the members of one of the parties always tell the truth, while all of the members of the other party always lie, but you don't know which party tells the truth and which party lies.

Suppose you meet a party member on the street, whose party affiliation cannot be determined by her clothes.

(a) By asking one yes-no question, find out whether she is a democrat or a republican.

(b) By asking one more yes-no question, find out whether it is the democrats or the republicans that tell the truth.

(Formalise both of your questions in propositional logic).

(c) Write an electrical circuit representing each of the questions you will pose in (a) and (b).

2. Exercise

In a certain country there are three kinds of people; workers (who always tell the truth), politicians (who always lie), and students (who sometimes tell the truth and sometimes lie). Suppose you are visiting this country and arrive at a fork in the road, where one branch leads to the capital and the other doesn't. There is no sign indicating which way the capital is, but at the fork there are three locals, a worker, a politician and a student, whose class cannot be determined by their appearance.

- By asking two yes-no questions, find out which branch leads to the capital (each question may be addressed to any one of the three).

3. EXERCISE

Write axioms for each of the following theories of the language $L = \{<\}$, where $<$ is a binary relation symbol:

(a) linear orderings,

(b) dense linear orderings,

(c) dense linear orderings with a left endpoint, but no right endpoint.

(d) linear orderings such that each point has an immediate predecessor and an immediate successor.

Consider now the language $L = \{<, f\}$, where $<$ is a binary relation symbol and f a unary function symbol

(e) Write axioms for the theory stating that $<$ is a linear ordering and f is a function associating to each element its immediate successor in the ordering.