Making Math Engaging
Discrete Mathematics for K-8 Teachers

Module 1: Vertex-Edge Graphs
Activity Book

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Module 1 – Vertex-Edge Graphs.....

Activity Book
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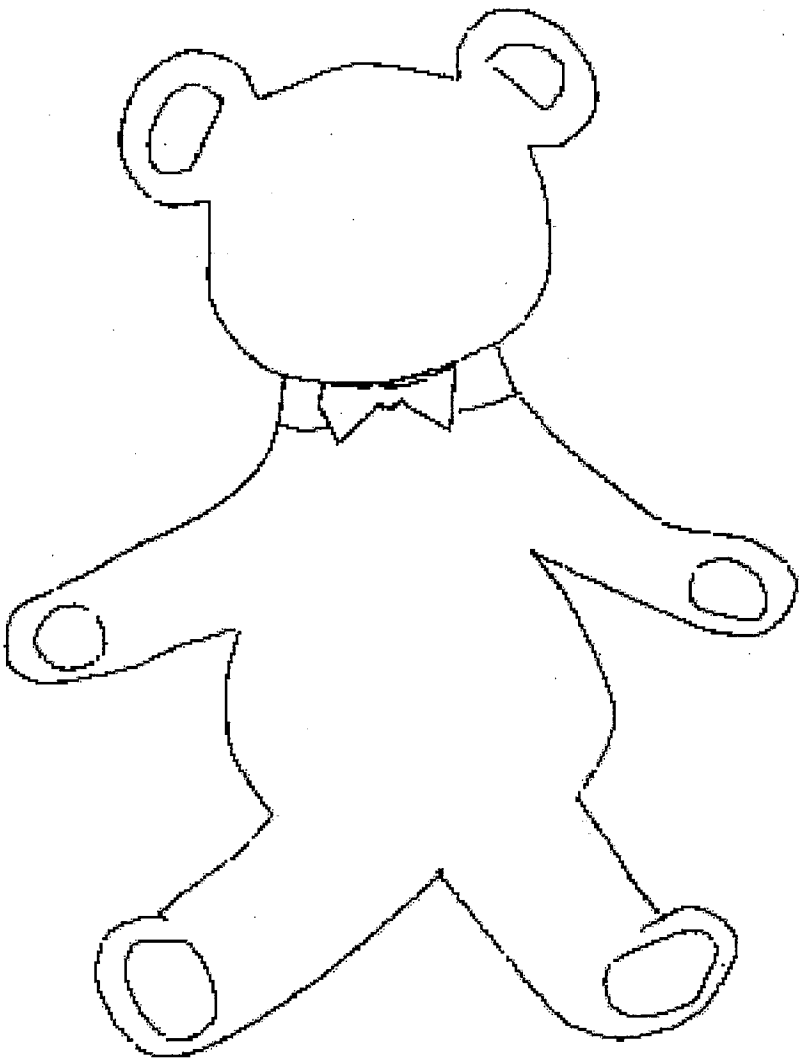
Activity 1 **An Illustrator's Dilemma**

Color the eleven regions of the bear using as few colors as possible. Remember that neighboring parts must have different colors.
How many colors did you use? _____



How do you know that you can't color the bear using fewer colors? _____

_____

_____

_____

Activity 2 **Coloring the Map of the Western States**

Can you color the twenty-two states west of the Mississippi River using a small number of colors? (Remember, if any two states share a border, they must have different colors.)

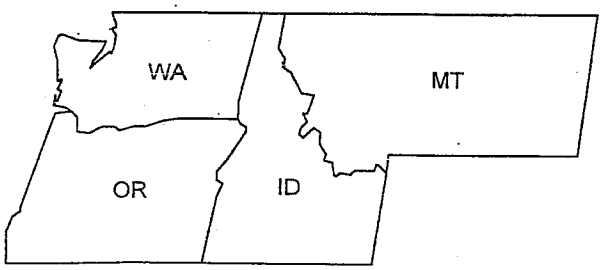
Since you will want to experiment with your colors, it might be simplest to pencil lightly in each state a letter R, W, B, P, Y, etc. (for red, white, blue, purple, yellow, etc.). Then, if you change your mind about what color to assign a state, you will be able to change it easily.

What's the smallest number of colors with which you were able to color the map? _____

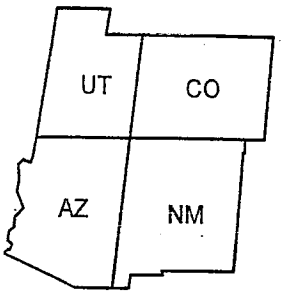
How do you know that you can't color this portion of the US map using fewer colors?

Activity 3 **Coloring Four Regions of the United States**

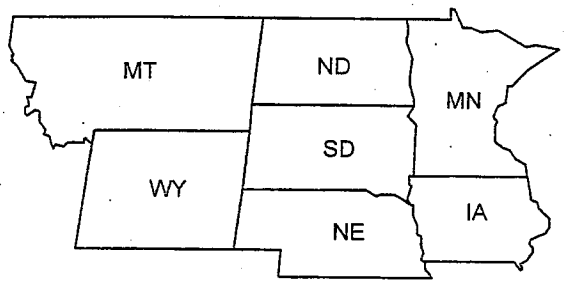
Can you color the states in each of these maps using three colors – say red, white, and blue? Remember, any two states that share a common border must have different colors so that you can tell where one state ends and the other begins. If you can't color a map using three colors, what is the fewest number of colors needed?



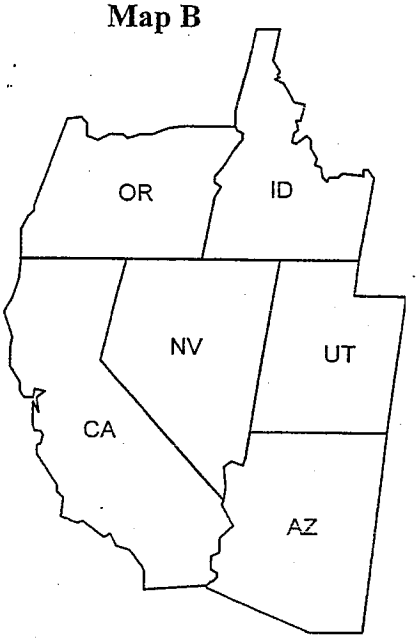
Map A



Map B



Map C



Map D

What is the smallest number of colors needed to color Map A? _____ Why can't you use one fewer color?_____

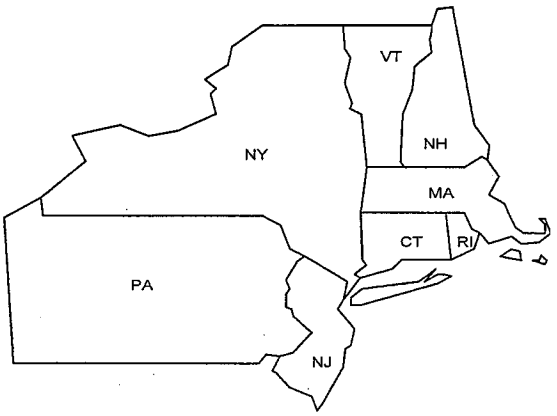
What is the smallest number of colors needed to color Map B? _____ Why can't you use one fewer color?_____

What is the smallest number of colors needed to color Map C? _____ Why can't you use one fewer color?_____

What is the smallest number of colors needed to color Map D? _____ Why can't you use one fewer color?_____

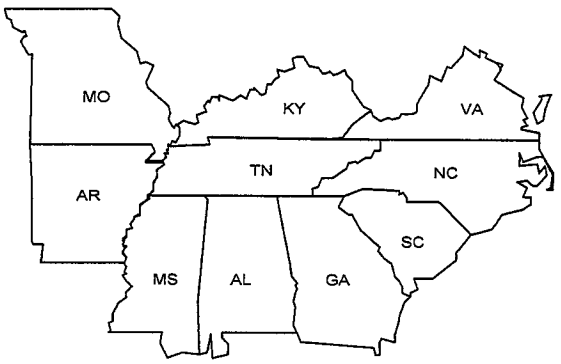
Activity 4 **Coloring Three More US Map Regions and a Bear**

Color each of the following regions and bear using as few colors as possible. Be sure to explain why one fewer color will not be sufficient.

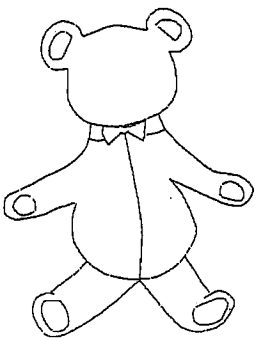


Map E

Map F



Map G



Bear Map

What is the smallest number of colors needed to color Map E? _____ Explain why one fewer color will not be sufficient._____

_____

What is the smallest number of colors needed to color Map F? _____ Explain why one fewer color will not be sufficient._____

_____

What is the smallest number of colors needed to color Map G? _____ Explain why one fewer color will not be sufficient._____

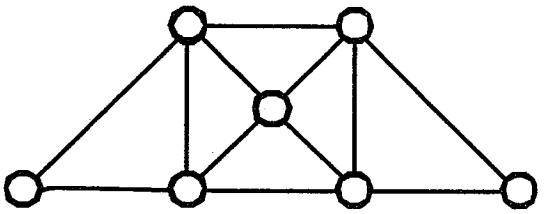
_____

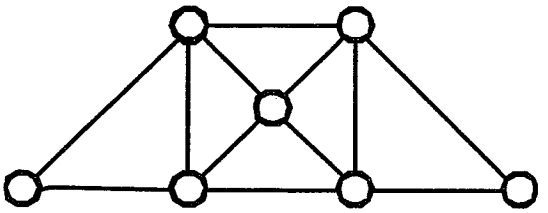
What is the smallest number of colors needed to color Bear Map? _____ Explain why one fewer color will not be sufficient._____

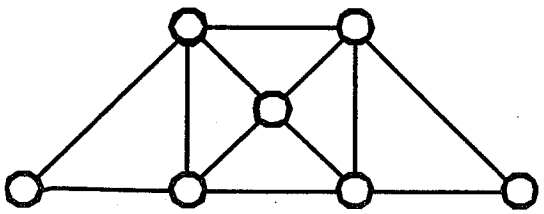
_____

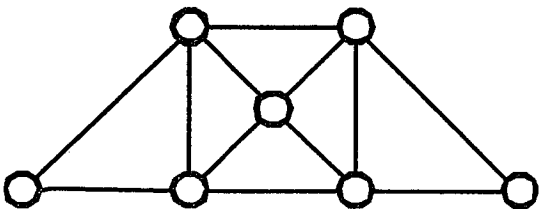
Activity 5 **Coloring a Graph**

What is the smallest number of colors needed to color the vertices of the graph below? Four copies are provided: one copy for your solution and three practice copies, if needed. Remember, two vertices that are neighbors must have different colors.







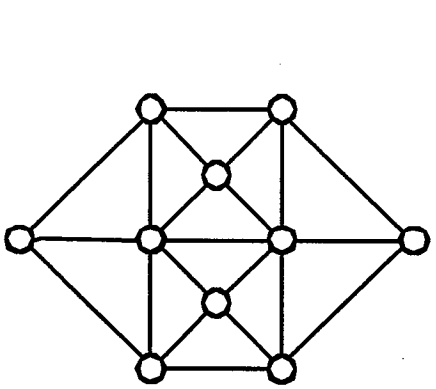


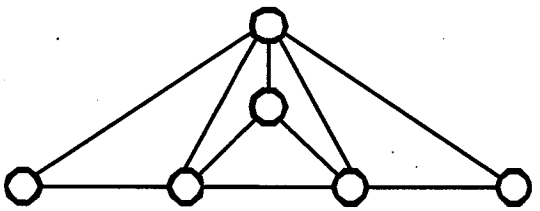
What is the smallest number of colors needed to color this graph? _____

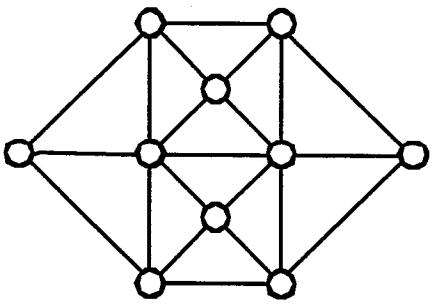
How do you know that you can't color this graph using fewer colors? _____

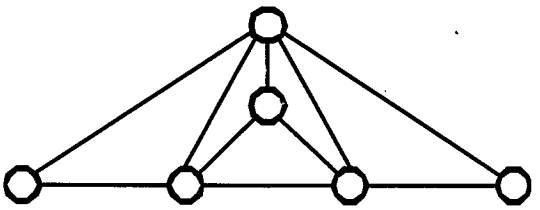
Activity 6 **Coloring Two Graphs**

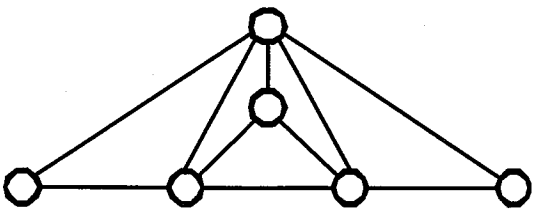
What is the smallest number of colors needed for a coloring of each of the following two graphs? (Several copies are provided of each graph.) Remember you have two tasks – find the smallest number of colors needed for each graph, and explain why you can't color each graph using fewer colors. Note that these graphs are similar to the graph in Activity 5. In the graph on the left, the Activity 5's graph has been reflected along its bottom border, and in the graph on the right, the top two vertices in Activity 5's graph have been pinched together into one vertex which essentially eliminates two edges. Can you identify these edges?











What is the smallest number of colors needed to color the graph at the left? _____

Why can't you color it using one fewer color?_____

_____

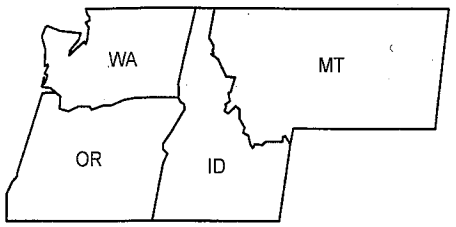
What is the smallest number of colors needed to color the graph at the right?_____

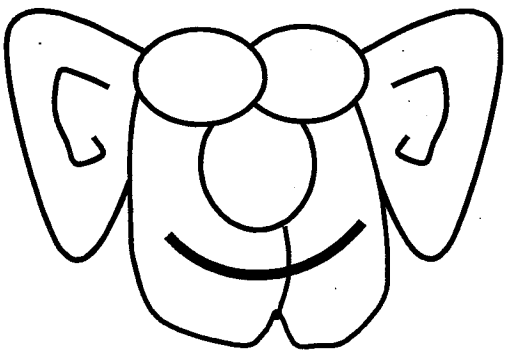
Why can't you color it using one fewer color?_____

_____

Activity 7 **Maps and Their Associated Graphs**

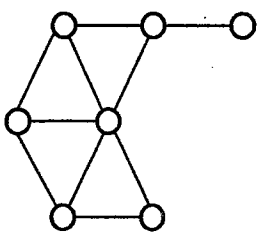
For each of the figures below, two familiar maps and one friendly face, draw its associated graph, color the graph using as few colors as possible, and then transfer the coloring back to the original map (or face).

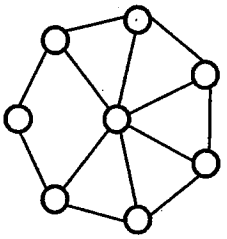


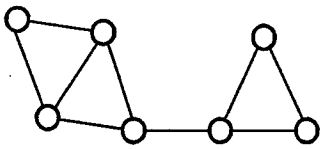


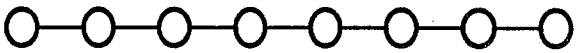
Activity 8 **Associated Graphs and Their Maps**

These four graphs are the graphs associated with four portions of the United States map. Can you label the vertices with the names of states so that two vertices are connected by an edge exactly if the corresponding states share a common border?









Activity 9 **Cycles, Wheels, and Paths**

1. For each set of states, draw a graph whose vertices are the states and whose edges connect states that share a common border, and determine whether or not the states in the set form a cycle.
   a. Texas, New Mexico, Colorado, Nebraska, Missouri, Tennessee, Mississippi, Louisiana
   b. Texas, New Mexico, Colorado, Nebraska, Missouri, Arkansas, Louisiana
   c. California, Nevada, Utah, Arizona
   d. Minnesota, Iowa, Nebraska, Wyoming, Montana, North Dakota.

2. For each of the following states, draw its neighbor graph and determine whether or not it is a wheel.
   a. Kansas
   b. Illinois
   c. Mississippi

3. What is the longest list of states whose associated graph is a path?

Activity 10 **Coloring Paths**

What is the least number of colors needed to color each path P_n?

Activity 11 **Coloring Complete Graphs and Wheels**

1. What is the chromatic number of a complete graph? That is, what is the least number of colors needed to color each complete graph K_n?

[Note: If you claim that N is the chromatic number of a graph, then you need to find a coloring of the graph using N colors and you need to explain why you can't color the graph with N–1 colors.]

2. What is the chromatic number of a wheel? That is, what is the least number of colors needed to color each wheel W_n?

[Note: If you claim that N is the chromatic number of a graph, then you need to find a coloring of the graph using N colors and you need to explain why you can't color the graph with N–1 colors.]

Activity 12 **Coloring the United States Map**

Can you color the United States map using four colors?

[Note: Don't actually color the states until you have found a solution with the least number of colors, since the colors are difficult to erase. You might want to just pencil in the letters R, B, G, Y, P, etc., instead of coloring red, blue, green, yellow, purple, etc. until you find a solution.]

Activity 13 **Wheeling Around America**

1. Which states are completely surrounded by other states, that is, every portion of their border is shared by another state in the United States?

2. Which of these states are the hubs of wheels with an odd number of spokes? Are there any others besides Nevada and Kentucky?

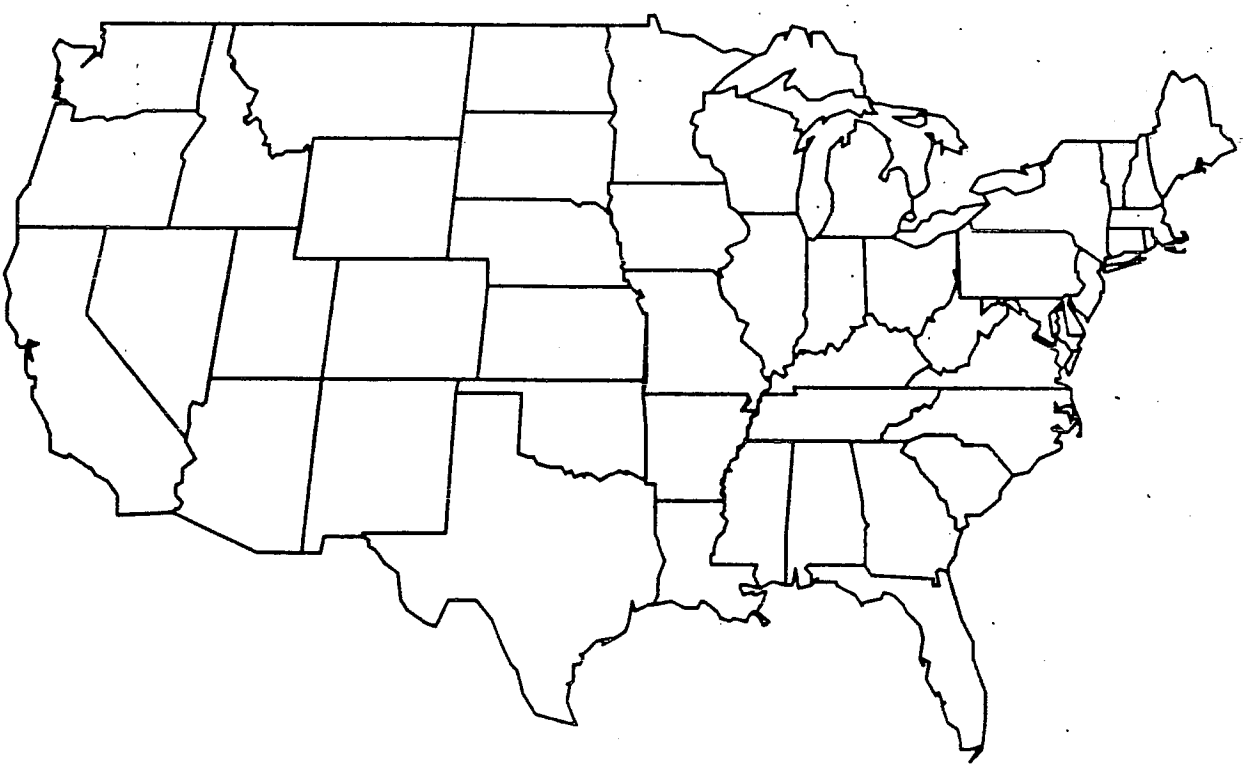
3. Which states are the hubs of wheels with an even number of spokes?

4. Which states are completely surrounded by other states but are not the hubs of wheels at all? Explain how this can happen.

Activity 14 **Coloring the United States Map ... Again**

1. Can you color Nevada and Kentucky yellow and all of the other states red, white, and blue?

2. Can you color the 48 states in the United States map using four colors so that each color is used exactly 12 times?



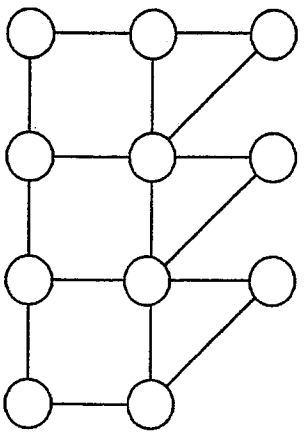
Activity 15 **"Complete Maps"**

1. Can you draw a map with four regions each of which borders the other three regions?

2. Can you draw a "different" map with four regions each of which borders the other three regions?

Activity 16 **Subgraphs**

1. For which n can you find a subgraph of this graph that has the same form as P_n?



2. For which n can you find a subgraph of this graph that has the same form as K_n?

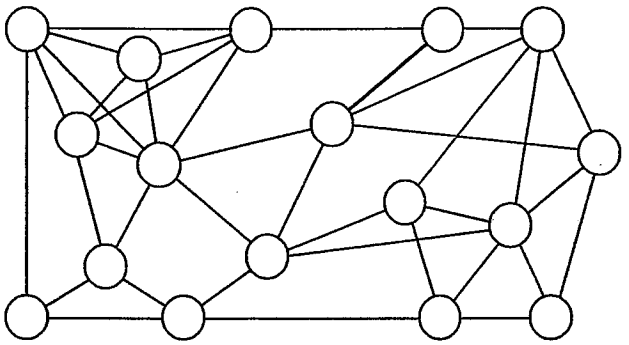
3. For which n can you find a subgraph of this graph that has the same form as C_n?

4. For which n can you find a subgraph of this graph that has the same form as W_n?

Activity 17 **Coloring a Graph**

What is the chromatic number of this graph? _____

Find a coloring of the graph using that many colors.

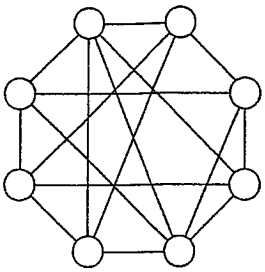


Explain why there is no coloring using fewer colors.

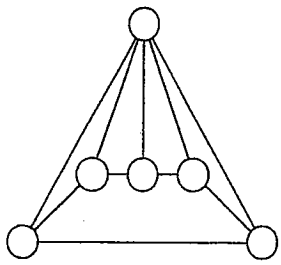
Activity 18 **Coloring More Graphs**

Find the chromatic number for each of the following seven graphs.

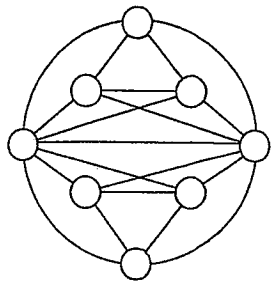
[Note: If you claim that N is the chromatic number of a graph, then you need to find a coloring of the graph using N colors and you need to explain why you can't color the graph with N – 1 colors.]



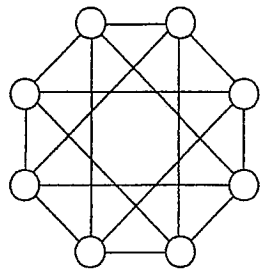
$\chi(G) =$ _____



$\chi(G) =$ _____

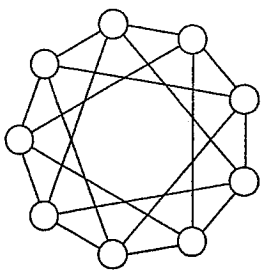


$\chi(G) =$ _____

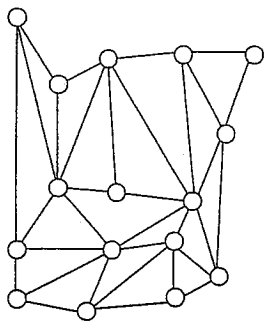


$\chi(G) =$ _____

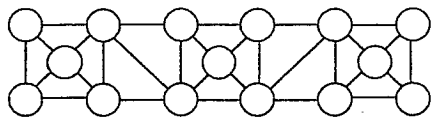
Why can't you use one fewer color?



$\chi(G) =$ _____



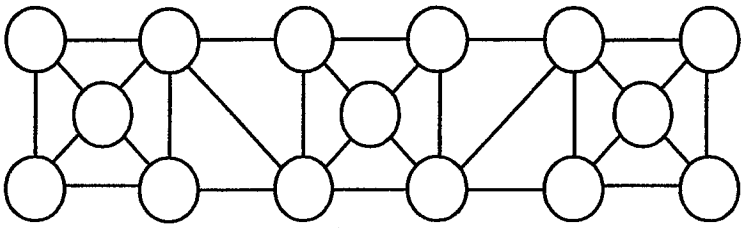
$\chi(G) =$ _____



$\chi(G) =$ _____

Why can't you use one fewer color?

Activity 19 **An Argument to Show "Why"**



This graph has 15 vertices and 30 edges, yet there is essentially only one way of coloring it using three colors. Can you develop an argument to convince someone that this fact is true?