Homework problems ##8 and 9
Math 536
Due Monday, Nov. 9, 2009

8. Prove that the family of functions

$$\left(\frac{1}{((m-\frac{1}{2})\tau+(n-\frac{1}{2}))^2}-\frac{1}{(m\tau+n)^2}\right)_{(0,0)\neq(m,n)\in\mathbf{Z}\times\mathbf{Z}}$$

is locally uniformly absolutely summable on the upper half-plane $\mathbf{H}$. (It was pointed out in class that this implies that the function e_3 is holomorphic on $\mathbf{H}$. Slightly simpler calculations, which you do NOT have to write down, show that e_1 and e_2 are holomorphic.)

9. **(a)** Prove the following proposition, which was stated in class. Let $f : X \to Y$ be a non-constant proper map between connected Riemann surfaces. Then for every point y the set $f^{-1}(y) \subset X$ is finite. Furthermore, the integer

$$d = \sum_{x\in f^{-1}(y)} (\text{loc deg}_x(f)),$$

where loc $\deg_x(f)$ denotes the local degree of f at x, is independent of the choice of the point $y \in Y$. (Adapt the proof given in class for the compact case. You will need the fact that a proper map is closed.)

The integer d is called the degree of f.

(b) Prove that under the above hypotheses there is a discrete, closed subset $S \subset X$ such that for every point $y \in Y \setminus f(S)$, the set $f^{-1}(y)$ has cardinality d. This is a corrected version of an assertion that I made in class.