

Math 180 Final Exam — Fall 2009

For each problem give a complete written solution which states the method being used and clearly shows all the mathematical steps. Numerical answers can be given in terms of constants such as e , π , $\ln 2$, $\sqrt{5}$.

1. Differentiate with respect to x . Write your answers showing the use of the appropriate techniques. Do **not** simplify.

(a) $(x^2 + 1)\sin(x)$, (b) $\frac{5x - 3}{x^2 + x + 1}$, (c) e^{x^2+3} .

2. (a) Write a definite integral that gives the area of the region between $x = 0$ and $x = 2$, above the x -axis, and below the curve $y = 4x - x^3$.
(b) Evaluate this integral to find the exact area.

3. Find $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$.

4. Find an antiderivative for $f(x) = x^3 + x^{-2} + x^{2/3}$, that is, find $\int (x^3 + x^{-2} + x^{2/3}) dx$.

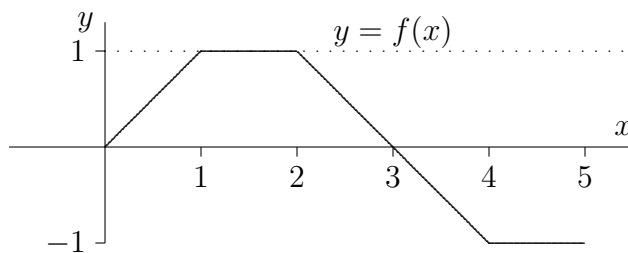
5. (a) Find the critical points of the function $f(x) = \frac{x - 1}{x^2 - x + 1}$, and classify each critical point as a local minimum, local maximum, or neither.

(b) Find the exact values of $f(x)$ at the critical points.

(c) The graph $y = f(x)$ has a horizontal asymptote $y = L$. What is the value of L ?

(d) Sketch the graph $y = f(x)$ using your answers to (b) and (c).

6. The graph of a function $f(x)$ is shown below.



(a) Find $\int_0^3 f(x) dx$.

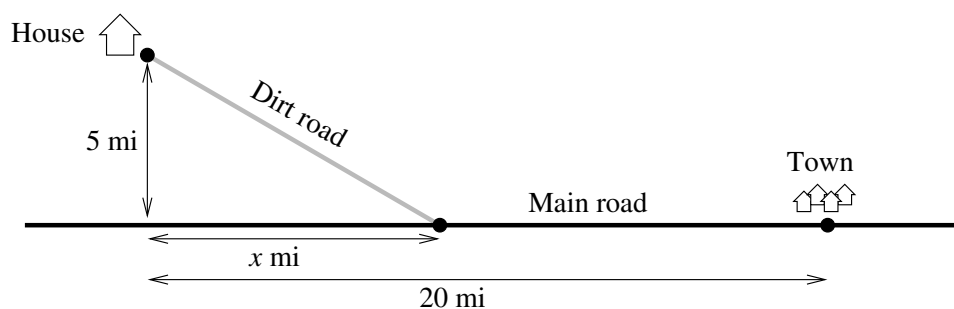
(b) Find $\int_0^5 f(x) dx$.

(c) If $F(x) = \int_0^x f(t) dt$, find $F'(2)$.

7. Evaluate the integral $\int \sin^3 x \cos x \, dx$.

8. Evaluate the integral $\int \frac{e^x}{1 + e^x} \, dx$.

9. A house lies 5 miles north of a paved road that runs east-west. The nearest town lies 20 miles east on the main road. The owner of the house wants to build a dirt road from the house to the main road that will meet the main road at point x miles east of the point due south of the house. A car can travel 30 miles per hour on the dirt road and 50 miles per hour on the main road. The goal of this problem is to find the value of x that will minimize the travel time from the house to the town.



- What is the distance from the house, at $(0, 5)$, to the point $(x, 0)$ on the main road?
- What is the distance from $(x, 0)$ to town, $(20, 0)$?
- What is the total travel time $T(x)$ for the trip from the house to the town as a function of x ? Recall that time = distance / speed.
- Find the value of x minimizing $T(x)$.