

**First Hour Exam**

(16 pts) **1.** Evaluate the following limits, or show that they do not exist

(a)  $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 - 4}$

(b)  $\lim_{x \rightarrow 3} \frac{|x^2 - 9|}{x^2 + 9}$

(c)  $\lim_{x \rightarrow 2} \frac{x - 2}{\sqrt{x} - \sqrt{2}}$

(16 pts) **2.** Consider the equation  $x^3 + x + 1$ .

- (a) Use the Intermediate Value Theorem to show that it has a solution in the interval  $[-2, 0]$ .  
(b) Use the Bisection Method to find an interval of length  $\frac{1}{2}$  that contains a solution.

(16 pts) **3.** Let  $f(x) = x^2 - 2x - 2$ .

- (a) Use the definition of the derivative as a limit of difference quotients to compute  $f'(3)$ .  
(b) Find an equation of the tangent line to the graph of  $f$  at the point  $(3, 1)$ .

(16 pts) **4.** Let  $f(x) = \frac{1}{1-x} + 3$ .

- (a) Find the average rate of change of the function between  $x = -0.6$  and  $x = -0.4$ .  
(b) Find the instantaneous rate of change at  $x = -0.5$ .

(20 pts) **5.** Find the derivatives of the following functions using the basic rules. Leave your answers in an unsimplified form so that it is clear what method you used.

(a)  $f(x) = \sin(x^3)$

(b)  $f(x) = x^2 \cdot \arctan(3x)$

(c)  $f(x) = \frac{1 - \cos x}{x^2 + 1}$

(d)  $f(x) = x^3 e^{-x}$ .

(16 pts) **6.** Find the value/s of  $c$  for which the function

$$f(x) = \begin{cases} x^2 + 3 & \text{if } x < 2 \\ cx - 1 & \text{if } x \geq 2 \end{cases}$$

is continuous at  $x = 2$ . Justify your answers.