

First Exam

(20 pts) **1.** Evaluate the following limits. Show your work.

(a) $\lim_{x \rightarrow 0} \frac{2 \cos x}{\sqrt{x+1} - 2}$

(b) $\lim_{x \rightarrow +\infty} \frac{2x^2 + 4x - 1}{x^3 + 1}$

(30 pts) **2.** Compute the derivatives of the following functions AND state where the derivative does not exist. Show your work and do not simplify your answers.

(a) $\frac{x^2 + 1}{x}$

(b) $|x|$

(c) $e^{\sin(3x)}$

(20 pts) **3.**(a) Find an equation of the tangent line at $x_0 = 1$ to the graph of the following function:

$$f(x) = x^4 - x^2 + 1$$

(b) Find all those points x_0 where the tangent line to the graph is horizontal. Show your work.

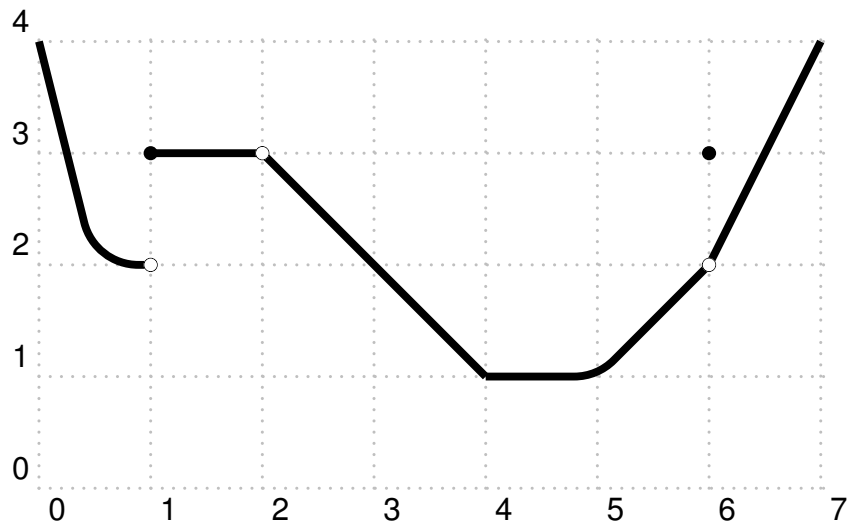
(10 pts) **4.** Use the Intermediate Value Theorem to show that there exists a solution to the equation

$$\cos x = x$$

on the interval $[0, \frac{\pi}{2}]$. Show your work.

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- (20 pts) 5. Consider the function f whose graph appears below, and answer the following questions. **You must justify all answers.**



- (a) (i) Is $f(1)$ defined? If so, what is it?
 (ii) Does $\lim_{x \rightarrow 1} f(x)$ exist? If so, what is it?
 (iii) Is f continuous at 1?
- (b) (i) Is $f(2)$ defined? If so, what is it?
 (ii) Does $\lim_{x \rightarrow 2} f(x)$ exist? If so, what is it?
 (iii) Is f continuous at 2?
- (c) (i) Is $f(4)$ defined? If so, what is it?
 (ii) Does $\lim_{x \rightarrow 4} f(x)$ exist? If so, what is it?
 (iii) Is f continuous at 4?
- (d) (i) Is $f(6)$ defined? If so, what is it?
 (ii) Does $\lim_{x \rightarrow 6} f(x)$ exist? If so, what is it?
 (iii) Is f continuous at 6?

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