MATH 180
Sample exam problems for the 2nd hour exam
Fall 2009

1. Differentiate the functions: (do not simplify)

$$f(x) = x\cos(x^2), \qquad f(x) = xe^{\sqrt{x}},$$
$$f(x) = \arcsin(3x+1) = \sin^{-1}(3x+1), \qquad f(x) = \frac{e^{3x}}{\ln x}.$$

2. The following table of values is provided for the functions f, g and their derivatives:

x	1	3
$f(x)$	2	4
$f'(x)$	1	5
$g(x)$	3	-2
$g'(x)$	2	-3

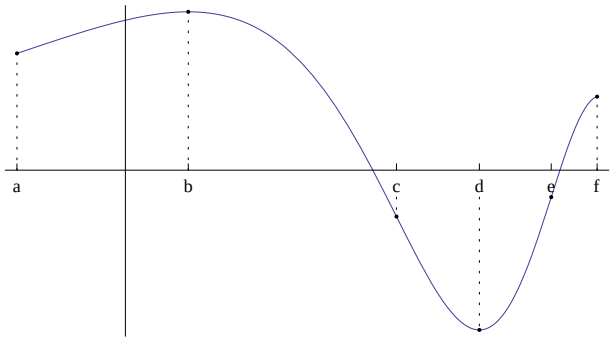
   Let $h(x) = f(g(x))$ and compute $h'(1)$.

3. Differentiate the functions: (do not simplify)

$$f(x) = \sin(x^2+5x+2), \quad f(x) = \ln(x+\cos x), \quad f(x) = (1+\ln x)^{3/4}.$$

4. Find the derivative of the function $y = x^x$.

5. The graph of a function $f(x)$ is given below:



   List the intervals on which f is increasing, decreasing, concave up, and concave down.

6. Find the equation of the tangent to the curve $y^2x + x + 2y = 4$ at the point $(1, 1)$.

7. Let $f(x) = xe^x$.

   (a) Find and classify the critical points of f.

   (b) Is there a global minimum of f over the entire real line? Why or why not?

8. Find the minimum and maximum of the function $f(x) = x^3 - 3x$ over the interval $[0, 2]$.

9. A function f is defined on $[0, 2]$ by $f(x) = x^2 + x + 1$ for $0 \leq x \leq 2$. Let g be the inverse function of f. Find $g'(3)$.

10. Find the limits

$$\text{(a) } \lim_{x \to 0} \frac{1 - \cos(3x)}{x^2} \qquad \text{(b) } \lim_{x \to \pi/6} \frac{1 - \cos(3x)}{x^2}.$$

11. Find

$$\lim_{x \to 0^+} x \ln x.$$

12. Find the critical points of the function $f(x) = x^3 + x^2 - x + 5$ and determine if they correspond to local maxima, minima, or neither.

13. Let $f(x) = x^4 + 2x^2$. Determine the intervals on which f is increasing or decreasing and on which f is concave up or down.

14. Let $f(x) = 2x^3 + 3x^2 - 12x + 1$.

   (a) Find the critical points of f.

   (b) Find the intervals on which f is increasing and the intervals on which f is decreasing.

   (c) Find the local minima and maxima of f. Compute x and $f(x)$ for each local extremum x.

   (d) Determine the intervals on which f is concave up and the intervals on which f is concave down.

   (e) Find the points of inflection of f.

   (f) Sketch the graph of f.

15. Use the Newton approximation method to estimate the positive root of the equation $x^2 - 2 = 0$. Begin with $x_0 = 2$ and compute x_1. Present your answer as a fraction with integer numerator and denominator.

16. A rectangle has its left lower corner at $(0, 0)$ and its upper right corner on the graph of
$$f(x) = x^2 + \frac{1}{x^2}$$
   (a) Express its area as a function of x.
   (b) For which x is the area minimum and what is this area?

17. A box has square base and total surface area equal to 12 m^2.
   (a) Express its volume as a function of x, the length of one side of the base.
   (b) Find the maximum volume of such a box.

18. You plan to build a wall enclosing a rectangular garden area of 20,000 square meters. There is a river on the one side, the wall will be built along the other 3 sides. Determine the dimensions that will minimumize the length of wall.

19. Find the linearization of the function $f(x) = \dfrac{1}{x^2 + 1}$ at the point $a = 1$.

20. Find antiderivatives of the functions:
$$\text{(a) } f(x) = 3x^2 - 2x \quad \text{(b) } f(x) = e^{3x} \quad \text{(c) } f(x) = \frac{2}{x^2} \quad \text{(d) } f(x) = \cos(2x)$$