MATH 180
2nd Hour Exam
A collection of problems

This is a list of problems to help you prepare for the 2nd hour exam. It is not considered to be exhaustive and you should not expect to find your actual exam problems in the list below. It is to serve as a study aid and it cannot be a substitute for in-class reviews or study of your class notes.

1. Let $f(x) = \dfrac{x}{x^2+1}$.
   - Determine the intervals on which f is increasing and those on which it is decreasing.
   - Determine the intervals on which f is concave up and those on which it is concave down.
   - Find the critical points of f and determine if they correspond to local extrema.
   - Find the asymptotes of f.
   - Determine the global extrema of f.
   - Sketch the graph of f.
2. Let $f(x) = xe^x$.
   i) Find and classify the critical points of f.
   ii) Find the global minimum of f over the entire real line.
3. Find the minimum and maximum of the function $f(x) = \sqrt{6x - x^3}$ over the interval $[0, 2]$.
4. Let $f(x) = 3x - x^3$.
   i) On what interval(s) is f increasing?
   ii) On what interval(s) is f decreasing?
   iii) On what interval(s) is f concave up?
   iv) On what interval(s) is f concave down?
   v) Sketch the graph of f
5. For a function $f(x)$ we know that $f(3) = 2$ and that $f'(3) = -3$. Give an estimate for $f(2.91)$.
6. Let $f(x) = \dfrac{x^2+1}{x+1}$. Find the best linear approximation of f around the point $x = 0$ and use it in order to estimate $f(.2)$. Would this be an underestimate or an overestimate?

7. A rectangular farm of total area 20,000 sq. feet is to be fenced on three sides. Find the dimensions that are going to give the minimum cost.

8. Let $f(x) = 3x^5 - x^3$.

   - Find the critical points of f.
   - Determine the intervals on which f is increasing and the ones on which it is decreasing.
   - Determine the intervals on which f is concave up an the ones on which it is concave down.
   - Determine the inflection points of f.
   - Sketch the graph of f.

9. A rectangle has its left lower corner at $(0, 0)$ and its upper right corner on the graph of

$$f(x) = x^2 + \frac{1}{x^2}$$

   i) Express its area as a function of x.

   ii) Determine x for which the area is minimum?

   iii) Can the area of such a rectangle be as large as we please?

10. A box has square base of side x, and constant surface equal to $12m^2$.

    i) Express its volume as a function of x.

    ii) Find the maximum volume of such a box.

11. Use the Newton approximation method in order to find x_2 as an estimate for the positive root of the equation $x^2 - 5 = 0$ when $x_o = 5$.

12. Use L' Hôpital's rule in order to compute the following limits:

$$\lim_{x\to 0} \frac{\ln(3x+1)}{\ln(5x+1)} \quad \lim_{x\to 0^+} x \ln x \quad \lim_{x\to 0} \frac{e^{3x}-1}{\tan x}$$

$$\lim_{x\to 4} \left(\frac{1}{\sqrt{x}-2} - \frac{4}{x-4} \right) \quad \lim_{x\to +\infty} \frac{e^x}{x + \ln x}$$

13. Compute the following indefinite integrals:

$$\int (x^2 - 5x + 6)dx \quad \int \sqrt[3]{x}(x^2 - \sqrt{x})dx \int e^{3x}dx$$

14. Consider the function $f(x) = x^2 - x$ on $[0, 2]$. Compute L_4 and R_4.

15. Use the Fundamental Theorem of Calculus in order to compute the following integrals:

$$\int_0^2 (x^2 + x + 1)dx \quad \int_1^4 \sqrt{x}dx \quad \int_0^\pi \sin(2x)dx$$