

MATH 181
Final exam
Sample problems

This is a list of problems to help you prepare for the final exam. It is not considered to be exhaustive and you should not expect to find your actual exam problems in the list below. It is to serve as a study aid and it cannot be a substitute for in-class reviews or study of your class notes.

1. Compute the definite integral:

$$\int_0^{\pi} x \cos(2x) dx$$

2. Find the following indefinite integrals:

$$\int \frac{x}{\sqrt{x-2}} dx \quad \int x^3 \sin(x^2) dx \quad \int \frac{dx}{x^2 + x - 6}$$

$$\int \frac{dx}{x^2 + x + 3} \quad \int \frac{dx}{x^3 - x} \quad \int x^6 \ln x dx$$

$$\int \arctan x dx \quad \int \cos(\sqrt{x}) dx \quad \int x^2 e^{2x} dx$$

3. Determine if the following improper integrals converge or not. If they converge, compute them:

$$\int_0^{+\infty} x e^{-2x} dx \quad \int_0^{+\infty} \frac{dx}{x^2 + 4} \quad \int_1^{+\infty} \frac{x^{3/2} + 3}{\sqrt{x}} dx \quad \int_0^1 \frac{dx}{\sqrt{x}} \quad \int_1^{+\infty} \frac{dx}{x^4}$$

4. Determine whether the following integrals converges or not:

$$\int_0^1 \frac{e^x}{\sqrt{x}} dx \quad \int_1^{+\infty} \frac{dx}{\sqrt{x^3 + x + 1}}$$

5. Compute the area enclosed between the graphs $y = 1 - x^2$ and $y = 1 - x$.
6. Find the center of mass of the region described in problem 5.
7. Let R be the region included by the curves $y = 0$ and $y = x^2 + x$ between $x = 0$ and $x = 1$. Find the volume of the solid of revolution when the axis is the line $y = -1$.
8. Find the arclength of the graph of the function $y = 2x^{3/2} + 5$ between $x = 0$ and $x = 1$.
9. Let $f(x) = x^2$ on the interval $[0, 1]$. Compute Mid(3) and Trap(3). Which one is an overestimate and why?
10. Find the sums of the following series:

$$\sum_{n=0}^{\infty} \frac{2^n - 1}{5^n} \quad \sum_{n=3}^{\infty} \frac{2 \cdot 3^{n-1}}{5^{n+2}}$$

11. Determine whether the following series converge or not:

$$\sum_{n=1}^{+\infty} \frac{n+2}{\sqrt{n^3+n+5}} \quad \sum_{n=1}^{+\infty} \frac{(n^2+1)3^n}{n!} \quad \sum_{n=2}^{+\infty} \frac{1}{n(\ln n)^3}$$

12. i) Show that the series

$$\sum_{n=1}^{+\infty} \frac{(-1)^n}{n2^n}$$

converges.

- ii) Let $S(k)$ be defined by:

$$S(k) = \sum_{n=1}^k \frac{(-1)^n}{n2^n}$$

Then $S(k)$ is the partial sum of the series. Compute k so that $S(k)$ is within .01 of the sum of the series.

13. Compute the 2nd Taylor polynomial of the function $x^3 - 3x^2 + 5x + 3$ centered at the point $a = 1$.
14. Compute the 3rd Taylor polynomial of the function $f(x) = \ln x$ centered at $a = 1$.
15. Estimate the error $|\cos 1 - T_4(1)|$ where $T_4(x)$ is the 4th Maclaurin polynomial of the function $\cos x$.
16. Compute the interval of convergence of the following power series:

$$\sum_{n=0}^{+\infty} \frac{(-2)^n (x+4)^n}{n+3} \quad \sum_{n=1}^{+\infty} \frac{3^n (x-1)^n}{n^2}$$