MATH 181
Final exam
Answer key

1. Compute the definite integral:

$$\int_0^\pi x\cos(2x)dx$$

Answer: 0.

2. Find the following indefinite integrals:

$$\int \frac{x}{\sqrt{x-2}}dx = \frac{2}{3}(x-2)^{3/2} + 4\sqrt{x-2} + C$$

$$\int x^3\sin(x^2)dx = -\frac{1}{2}x^2\cos(x^2) + \frac{1}{2}\sin(x^2) + C$$

$$\int \frac{dx}{x^2+x-6} = \frac{1}{5}\ln\left|\frac{x-2}{x+3}\right| + C$$

$$\int \frac{dx}{x^2+x+3} = \frac{2}{\sqrt{11}}\tan^{-1}(\frac{2}{\sqrt{11}}(x+1/2)) + C$$

$$\int \frac{dx}{x^3-x} = -\ln|x| + \frac{1}{2}\ln|x-1| + \frac{1}{2}\ln|x+1| + C$$

$$\int x^6\ln x dx = \frac{x^7}{7}\ln x - \frac{x^7}{49} + C$$

$$\int \arctan x dx = x\tan^{-1}x - \frac{1}{2}\ln(1+x^2) + C$$

$$\int \cos(\sqrt{x})dx = 2\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x} + C$$

$$\int x^2e^{2x}dx = \frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$$

3. Determine if the following improper integrals converge or not. If they converge, compute them:

$$\int_0^{+\infty} xe^{-2x}dx = \frac{1}{4}$$

$$\int_0^{+\infty} \frac{dx}{x^2+4} = \frac{\pi}{4}$$

$$\int_1^{+\infty} \frac{x^{3/2}+3}{\sqrt{x}}dx = divergent$$

$$\int_0^1 \frac{dx}{\sqrt{x}} = 2$$

$$\int_1^{+\infty} \frac{dx}{x^4} = \frac{1}{3}$$

4. Determine whether the following integrals converges or not:

$$\int_0^1 \frac{e^x}{\sqrt{x}}dx \qquad \int_1^{+\infty} \frac{dx}{\sqrt{x^3+x+1}}$$

Yes, Yes.

5. Compute the area enclosed between the graphs $y = 1-x^2$ and $y = 1-x$.

$$1/6$$

6. Find the center of mass of the region described in problem 5.

$$(1/2, 3/5)$$

7. Let R be the region included by the curves $y = 0$ and $y = x^2 + x$ between $x = 0$ and $x = 1$. Find the volume of the solid of revolution when the axis is the line $y = -1$.

$$\pi\frac{37}{10}$$

8. Find the arclength of the graph of the function $y = 2x^{3/2} + 5$ between $x = 0$ and $x = 1$.

$$\frac{2}{27}(10^{3/2} - 1).$$

9. Let $f(x) = x^2$ on the interval $[0, 1]$. Compute Mid(3) and Trap(3). Which one is an overestimate and why?

$$M_3 = \frac{35}{108}$$

$$T_3 = \frac{19}{54}\,(overestimate)$$

10. Find the sums of the following series:

$$\sum_{n=0}^{\infty} \frac{2^n - 1}{5^n} = \frac{5}{12} \qquad \sum_{n=3}^{\infty} \frac{2 \cdot 3^{n-1}}{5^{n+2}} = \frac{9}{625}$$

11. Determine whether the following series converge or not:

$$\sum_{n=1}^{+\infty} \frac{n+2}{\sqrt{n^3 + n + 5}} \qquad \sum_{n=1}^{+\infty} \frac{(n^2+1)3^n}{n!} \qquad \sum_{n=2}^{+\infty} \frac{1}{n(\ln n)^3}$$

No, Yes, Yes

12. i) Show that the series

$$\sum_{n=1}^{+\infty} \frac{(-1)^n}{n2^n}$$

converges. By the root test.

ii) Let $S(k)$ be defined by:

$$S(k) = \sum_{n=1}^{k} \frac{(-1)^n}{n2^n}$$

Then $S(k)$ is the partial sum of the series. Compute k so that $S(k)$ is within .01 of the sum of the series.

$$k = 4$$

13. Compute the 2nd Taylor polynomial of the function $x^3 - 3x^2 + 5x + 3$ centered at the point $a = 1$.

$$6 + 2(x - 1)$$

14. Compute the 3rd Taylor polynomial of the function $f(x) = \ln x$ centered at $a = 1$.

$$(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3.$$

15. Estimate the error $|\cos 1 - T_4(1)|$ where $T_4(x)$ is the 4th Maclaurin polynomial of the function $\cos x$.

$$\leq \frac{1}{5!}$$

16. Compute the interval of convergence of the following power series:

$$\sum_{n=0}^{+\infty} \frac{(-2)^n (x+4)^n}{n+3} \qquad \sum_{n=1}^{+\infty} \frac{3^n (x-1)^n}{n^2}$$

First: $|x+4| < \frac{1}{2}$, diverges at left-end point, converges conditionally at right-end point. Second: $|x-1| < \frac{1}{3}$, converges absolutely at both ends.