Math 181 – Fall 2011 Name: ____________________

Final Exam

Please do not write solutions on this sheet. Show all your work to get full credit.

(20 pts) **1.** Find the limit of the following sequences as $n \to \infty$:

(a) $a_n = \dfrac{3n^4 - n^3 + 2}{2n^4 + n^2 - 10}$

(b) $b_n = \dfrac{n + \sin(n)}{2n^2 - n + 1}$

(20 pts) **2.** Determine whether each series converges or diverges. Justify your answer.

(a)

$$\sum_{k=2}^{\infty} \frac{1}{k \ln k}$$

(b)

$$\sum_{k=1}^{\infty} \frac{(-1)^k k^2}{2^k}$$

(20 pts) **3.** Determine whether each improper integral converges or diverges. Justify your answer.

(a)

$$\int_0^2 \frac{1}{\sqrt{x}(x-2)}\, dx$$

(b)

$$\int_1^{\infty} \frac{\arctan x}{x^2}\, dx$$

(30 pts) **4.** Evaluate the following integrals

(a)

$$\int (\cos x)^{-1} \sin^3 x\, dx$$

(b)

$$\int \frac{1}{x^2 - 4x - 12}\, dx$$

(20 pts) **5.** Find the volume of the solid obtained by rotating about the x-axis the region enclosed by the graphs of $y = 2x - x^2$ and $y = x$.

(30 pts) **6.** Find the power series representation centered at 0 for the following functions. Give the interval of convergence of the series.

(a) $f(x) = \dfrac{1}{(1-x)^2}$

(b) $g(x) = x^2 e^{-x}$

The exam CONTINUES on the back of this page!

 Name: ____________________

(30 pts) **7.** Let $f(x) = \cos(2x) - 1 + 2x^2$.

(a) Find the first two non-zero terms in the Maclaurin series expansion of f.

(b) Using the expansion found in step (a) compute the limit

$$\lim_{x\to 0} \frac{\cos(2x) - 1 + 2x^2}{x^4}$$

(30 pts) **8.** An equation of a curve in polar coordinates is given by

$$r = 2\cos\theta, \quad 0 \le \theta \le 2\pi.$$

(a) Rewrite the equation in Cartesian coordinates. Sketch and identify the curve.

(b) Find the arc length of the curve using the integral formula.

Extra Credit(10 pts) Compute the following indefinite integral:

$$\int e^{\sin x}(x\cos x - \tan x \sec x)\, dx$$

Please enter the following information to identify your class:

	Time	Room	Instructor
Lecture			
Recitation			

Hand in this sheet along with your exam booklet!