MATH 181
A collection of problems

1. Differentiate the function:

$$f(x) = \int_{\sqrt{x}}^{x^2} e^{t^3} dt$$

2. Compute the definite integral:

$$\int_0^1 xe^{3x} dx$$

3. Find the following indefinite integrals:

$$\int \frac{x}{\sqrt{x-2}} dx \quad \int x^3 \sin(x^2) dx \quad \int \frac{dx}{x^2+x-6}$$
$$\int \frac{dx}{x^2+x+3} \quad \int \frac{dx}{x^3-x} \quad \int x^6 \ln x dx$$
$$\int \arctan x dx \quad \int \cos(\sqrt{x}) dx \quad \int x^2 e^{2x} dx$$

4. Determine if the following improper integrals converge or not.

$$\int_0^{+\infty} xe^{-2x} dx \quad \int_0^{+\infty} \frac{dx}{x^2+4} \quad \int_0^1 \frac{e^x}{\sqrt{x}} dx \quad \int_1^{+\infty} \frac{x^{3/2}+3}{\sqrt{x}} dx$$

5. Let R be the region included by the curves $y = 0$ and $y = x^2 + x$ between $x = 0$ and $x = 1$. Find the volume of the solid of revolution when the axis is the line $y = -1$.

6. Find the arclength of the graph of the function $y = 2x^{3/2} + 5$ between $x = 0$ and $x = 1$.

7. Let $f(x) = x^2$ on the interval $[0, 1]$. Compute Mid(3) and Trap(3). Which one is an overestimate and why?

8. Find the sums of the following series:

$$\sum_{n=0}^{+\infty} \frac{2^n - 1}{5^n} \quad \sum_{n=3}^{+\infty} \frac{2 \cdot 3^{n-1}}{5^{n+2}}$$

9. Determine whether the following series converge or not:

$$\sum_{n=1}^{+\infty} \frac{n+2}{\sqrt{n^3+n+5}} \quad \sum_{n=1}^{+\infty} \frac{(-1)^{n+1}(n^3+1)}{2^n} \quad \sum_{n=1}^{+\infty} \frac{(n^2+1)3^n}{n!} \quad \sum_{n=2}^{+\infty} \frac{1}{n \ln n}$$

10. i) Show that the series

$$\sum_{n=1}^{+\infty} \frac{(-1)^n}{n2^n}$$

converges.

ii) Let $S(k)$ be defined by:

$$S(k) = \sum_{n=1}^{k} \frac{(-1)^n}{n2^n}$$

Then $S(k)$ is the partial sum of the series. Compute k so that $S(k)$ is within .01 of the sum of the series.

11. Compute the interval of convergence of the following power series:

$$\sum_{n=0}^{+\infty} \frac{(-2)^n(x+4)^n}{n+3} \quad \sum_{n=0}^{+\infty} \frac{3^n(x-1)^{2n}}{n^2}$$

12. Find the 5th Taylor polynomial of the function $f(x) = \sin(2x)$ centered at $x = 0$.

13. Expand the function $f(x) = \sin(3x^2)$ into a power series and use its expansion in order to compute $f^{(39)}(0)$.

14. Determine the function which is represented by the power series:

$$\sum_{n=1}^{+\infty} nx^n$$

Use your answer to compute the sum:

$$\sum_{n=1}^{+\infty} n\left(\frac{2}{3}\right)^n$$