

Second Hour Exam

- (20 pts) **1.**(a) Find the Taylor polynomial $T_2(x)$ of degree 2 for $\ln(x)$ at $a = 1$.
(b) Use the Taylor polynomial from part (a) to estimate $\ln(1.1)$.
(c) Use the Error Bound to show that $|T_2(1.1) - \ln(1.1)| \leq \frac{1}{3000}$.
- (15 pts) **2.** Find the area of the surface of revolution of $y = 2x - 1$ about the x -axis over the interval $1 \leq x \leq 3$.
- (20 pts) **3.**(a) Show that $\int_1^{+\infty} \frac{dx}{x^7 + 1}$ converges.
(b) Show that $\int_9^{+\infty} \frac{dx}{\sqrt{x} - 2}$ diverges.
(c) Determine whether $\int_0^{+\infty} e^{-x^4} dx$ converges or diverges. Justify your answer.
- (15 pts) **4.** Find the centroid of the semicircular region bounded by $y = \sqrt{4 - x^2}$ and the x -axis.
- (20 pts) **5.** Determine whether each series is convergent or divergent. Justify your answers.
(a) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{e^n}$
(b) $\sum_{n=3}^{\infty} \frac{n^2 - 7}{n^2 + 1}$
- (10 pts) **6.** Compute the sum of each series. Express each answer in the form $\frac{a}{b}$ where a, b are integers.
(a) The geometric series $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \dots$
(b) $\sum_{n=0}^{\infty} \frac{2^{3n}}{3^{2n+3}}$