Math 210, Final Exam – Fall 2010 Name: ____________________

Final Exam

(20 pts) **1.** Let $\mathbf{u} = \langle 1, -1, 0\rangle$ and $\mathbf{v} = \langle 2, 1, 3\rangle$.
(a) Is the angle between $\mathbf{u}$ and $\mathbf{v}$ acute, obtuse, or right?
(b) Find an equation for the plane through the point $(1, -1, 2)$ containing $\mathbf{u}$ and $\mathbf{v}$.

(25 pts) **2.** The curve $\mathbf{r}(t) = \langle 2\sin(t), 2\cos(t), -t\rangle$ describes the movement of a particle in $\mathbb{R}^3$.
(a) Find the velocity and the acceleration of the particle as a function of t.
(b) Find the tangent line to the curve at time $t = \pi/4$.
(c) Find the distance travelled between time $t = 0$ and $t = \pi$.

(20 pts) **3.** Use Green's theorem to compute $\oint_C y\,dx + x^2 y\,dy$ where C traces the triangle with vertices $(0, 0)$, $(2, 0)$, $(1, 1)$ traversed in this order.

(25 pts) **4.** Find the critical points of $z = x^3 + x^2 + y^2 - 2xy - 12x$ and use the second derivative test to classify them as local maxima, local minima or saddles.

(25 pts) **5.** Consider the vector field $\mathbf{F} = \langle cx^2y^2 - e^y, 2x^3y - xe^y\rangle$ on $\mathbb{R}^2$ where c is a constant.
(a) Find the value for c that makes $\mathbf{F}$ a conservative vector field.
(b) With c as in (a) find a function $\phi(x, y)$ so that $\mathbf{F} = \nabla\phi$.

(25 pts) **6.** Compute the volume of the region in $\mathbb{R}^3$ bounded by the paraboloid $z = x^2 + y^2$, the cylinder $x^2 + y^2 = 9$, and the plane $z = 0$.

(20 pts) **7.** Given the function $f(x, y) = x\,y^2 + y\cos(x)$ find:
(a) the gradient ∇f at the point $P = (0, 1)$,
(b) the directional derivative $D_{\mathrm{v}} f(0, 1)$, where $\mathbf{v}$ is the unit vector from $P = (0, 1)$ towards $Q = (2, 3)$.

(25 pts) **8.** Use the method of Lagrange multipliers to find points where $f(x, y) = xy$ attains its maximum and minimum subject to the constraint: $x^2 + 4y^2 = 2$.

(15 pts) **9.** Given the function $f(x, y) = xe^{xy}$ compute the partial derivatives:

$$\frac{\partial f}{\partial x}, \quad \frac{\partial f}{\partial y}, \quad \frac{\partial^2 f}{\partial x\,\partial x}, \quad \frac{\partial^2 f}{\partial x\,\partial y}, \quad \frac{\partial^2 f}{\partial y\,\partial y}.$$

Hand in this sheet along with your exam booklet!