**MATH 210** **Final Exam** **May 7, 2009**

Name: ____________________

1. Let $f(x,y,z) = (x^2+y)z + x\cos(y^2-z)$.
   (a) Find the gradient $\nabla f$ at the point $P = (0,1,1)$.
   (b) Find the directional derivative $D_{\mathbf{v}}f(0,1,1)$ where $\mathbf{v}$ is the *unit vector* from $P$ towards $Q = (2,3,0)$.

2. Consider the vector fields $\mathbf{F} = \langle y\,e^{xy} + y^2,\, x\,e^{xy} + 2xy\rangle$ and $\mathbf{G} = \langle xe^{xy},\, ye^{xy}\rangle$.
   (a) Which of the two vector fields is conservative and which is not? (justify)
   (b) Find a potential $\phi$ for the conservative among the vector fields.

3. Use Green's theorem to compute

$$\oint_C xy^2\,dx + (x-y)\,dy$$

   where $C$ traces the triangle with vertices $(0,0)$, $(1,0)$, $(0,2)$ traversed in this order.

4. Let $\mathbf{u} = \langle 1,2,3\rangle$ and $\mathbf{v} = \langle 2,-1,0\rangle$.
   (a) What can be said about the angle between $\mathbf{u}$ and $\mathbf{v}$: acute/obtuse/right?
   (b) Find an equation for the plane through $(1,1,1)$ containing $\mathbf{u}$ and $\mathbf{v}$.

5. Find the equation of the tangent plane to the level surface $e^{xz} + (x+y)^3 - yz = 3$ at the point $(0,2,3)$.

6. Use the method of Lagrange multipliers to find points where $f(x,y) = x + 6y - 7$ attains its maximum and minimum on the ellipse $x^2 + 3y^2 = 13$.

7. Find all the critical values of $f(x,y) = x^3 + 2xy - 2y^2 - 10x$ and classify them into local maxima, local minima, and saddle points.

8. Let $C$ be the curve parametrized by $\mathbf{c}(t) = \langle 3t, 2\cos(t), 2\sin(t)\rangle$ for $0 \le t \le 2\pi$.
   (a) Find $\mathbf{c}'(t)$ and $\mathbf{c}''(t)$.
   (b) Find the length of the curve.

9. Let $H$ be the upper semi-ball $x^2+y^2+z^2 \le 4$, $z \ge 0$. Compute

$$\int\int\int_H z\,dV.$$

10. Change the order of integration and evaluate the iterated integral

$$\int_0^1 \int_{y^{1/3}}^1 (xy + \sin(x^4))\,dx\,dy.$$