Math 210 **Final Exam** **May 6, 2010**

For every question, write your solution with computations in the exam booklet. Each part of each problem is worth a certain number of points, indicated in the right margin.

1. The position vector [25 pts]

$$\vec{r}(t) = t^3\vec{i} + 18t\vec{j} + 3t^{-1}\vec{k}, \quad 1 \le t \le 2$$

describes the motion of a particle.

(a) Find the position at time $t = 2$.

(b) Find the velocity at time $t = 2$.

(c) Find the acceleration at time $t = 2$.

(d) Find the length of the path travelled by the particle during the time $1 \le t \le 2$.

2. (a) For $f(x, y) = e^{(x+1)y}$, find the derivatives: [20 pts]

$$\frac{\partial f}{\partial x}, \quad \frac{\partial f}{\partial y}, \quad \frac{\partial^2 f}{\partial x^2}, \quad \frac{\partial^2 f}{\partial x \partial y}, \quad \frac{\partial^2 f}{\partial y^2}$$

(b) Find the gradient of f at the point $(2, 3)$, $\nabla f(2, 3)$. [5 pts]

3. (a) Find a potential function for the vector field [10 pts]

$$\vec{F}(x, y, z) = (1 - z)\vec{i} + y\vec{j} - x\vec{k}.$$

(b) Integrate $\vec{F}$ over the straight line from (1,0,1) to (0,1,2). [10 pts]

[You may calculate this directly or you may use a potential function.]

4. (a) Find the critical points of the function $f(x, y) = x^3 - 3x - y^2$. [10 pts]

(b) Use the second derivative test to classify each critical point as a local maximum, local minimum, or saddle. [10 pts]

5. Find the maximum and minimum of the function $f(x,y) = (x-1)^2 + y^2$ [25 pts] subject to the constraint:

$$g(x,y) = \left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 1.$$

6. Compute the integral [20 pts]

$$\iint_R xy\,dx\,dy$$

over the quarter circle $R = \{(x,y) : 0 \le x,\ 0 \le y,\ x^2 + y^2 \le 1\}$

[You may use polar or Cartesian coordinates.]

7. Compute the integral [25 pts]

$$\iiint_R 1\,dx\,dy\,dz,$$

over the tetrahedron

$$R = \{(x,y,z) : 0 \le x,\ 0 \le y,\ 0 \le z,\ x/3 + y/5 + z/7 \le 1\}.$$

8. Find an equation for the tangent plane to the surface defined by $xy^2 + 2z^2 = 12$ at the point $(1,2,2)$. [20 pts]

9. Compute the integral [20 pts]

$$\oint (3x^2 + y)\,dx + (x^2 + y^3)\,dy$$

over the counterclockwise boundary of the rectangle

$$R = \{(x,y) : 0 \le x \le 3,\ 0 \le y \le 2\}$$

using Green's theorem or otherwise.