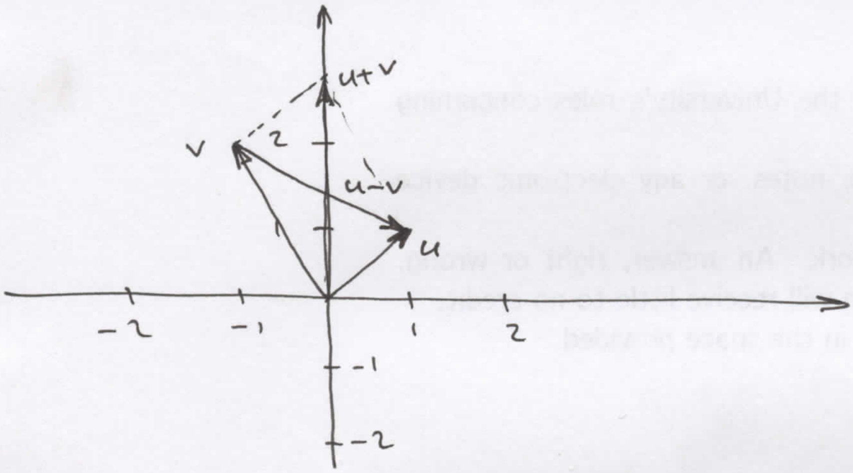
(6 pts) **1.** Two vectors are given by $\mathbf{u} = \langle 1, 1\rangle$, $\mathbf{v} = \langle -1, 2\rangle$. Draw the vectors on a coordinate grid. Find coordinates of the vectors $\mathbf{u}+\mathbf{v}$, $\mathbf{u}-\mathbf{v}$ and plot this couple of vectors on the same grid.



$u+v = \langle 1-1, 1+2\rangle$
$= \langle 0, 3\rangle$

$u-v = \langle 1+1, 1-2\rangle$
$= \langle 2, -1\rangle$

(9 pts) **2.** For each of the couples below determine whether the angle between the vectors is acute, obtuse, or right?

(a) $\mathbf{u} = \langle 3, 1, 2\rangle$, $\mathbf{v} = \langle -1, 0, 1\rangle$.

(b) $\mathbf{u} = 4\mathbf{i} - 2\mathbf{j}$, $\mathbf{v} = -\mathbf{j} + \mathbf{k}$.

(c) $\mathbf{u} = \langle -\frac{1}{2}, 3\rangle$, $\mathbf{v} = \langle 2, \frac{1}{3}\rangle$.

If $u \cdot v = 0$ the angle is right

$u \cdot v > 0$ —— " —— acute

$u \cdot v < 0$ —— " —— obtuse

(a) $u \cdot v = -3 + 0 + 2 = -1 < 0$ obtuse

(b) $u \cdot v = 4 \cdot 0 - 2 \cdot (-1) + 0 \cdot 1 = 2 > 0$ acute

(c) $u \cdot v = -1 + 1 = 0$ right.

(10 pts) **3.** Find an equation of the line perpendicular to two vectors $\mathbf{u} = \langle 1, 1, 4 \rangle$, $\mathbf{v} = \langle 0, -1, 2 \rangle$ and passing through the point $P(0, 1, 3)$.

Direction vector : $w = u \times v$

$$w = \begin{vmatrix} i & j & k \\ 1 & 1 & 4 \\ 0 & -1 & 2 \end{vmatrix} = \langle 6, -2, -1 \rangle$$

So, $r(t) = r_0 + tw$

$$x = 6t$$
$$\therefore \quad y = -2t + 1$$
$$z = -t + 3$$

(10 pts) **4.** Find an equation of the tangent line to the curve $\mathbf{r}(t) = \langle \frac{2}{t}, t, 2 \rangle$ at the point $t = 1$.

$$r'(t) = \left\langle -\frac{2}{t^2}, 1, 0 \right\rangle$$

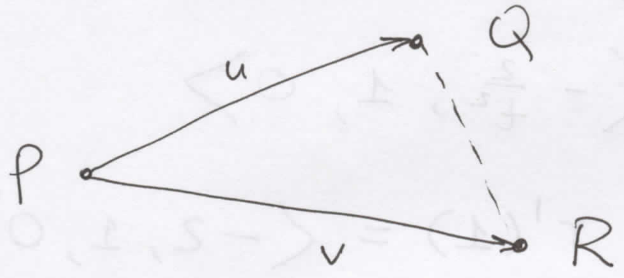
Direction vector $= r'(1) = \langle -2, 1, 0 \rangle$

The line passes through $r(1) = (2, 1, 2)$

$\therefore$

$$x = -2t + 2$$
$$y = t + \underline{1}$$
$$z = 2$$

(15 pts) **5.** Find the area of the triangle with vertices at $P(1, 0, 2)$, $Q(3, 1, 0)$, $R(0, 0, 2)$.



$$\text{Area} = \frac{1}{2}\,|u \times v|$$

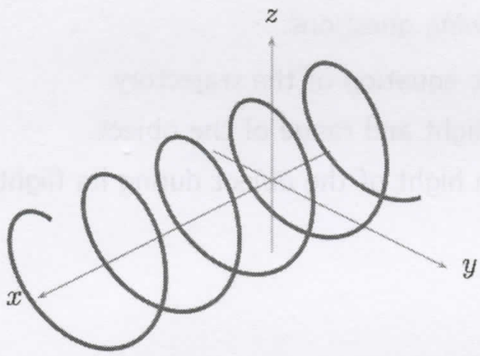
$$u = \langle 2, 1, -2 \rangle$$

$$v = \langle -1, 0, 0 \rangle$$

$$u \times v = \langle 0, 2, 1 \rangle$$

$$|u \times v| = \sqrt{4 + 1} = \sqrt{5}$$

$$\therefore \quad A = \frac{\sqrt{5}}{2}$$



(5 pts) **6.** Determine which one of these equations fits best for the curve pictured above. Explain your reasons, do not just give a guess.

(a) $\mathbf{r}(t) = \langle \cos t, \sin t, 3t \rangle$,

(b) $\mathbf{r}(t) = \langle \cos t, \sin t \rangle$,

(c) $\mathbf{r}(t) = \langle 2t, \sin t, \cos t \rangle$,

(d) $\mathbf{r}(t) = \langle t, 1 + t, -3t \rangle$.

The curve is a helix that projects onto a circle on the yz-plane as X goes off to infinity.

(a) no, because it is a circle on the xy-plane

(b) no, it is in $\mathbb{R}^2$!

(d) no, it is a line !

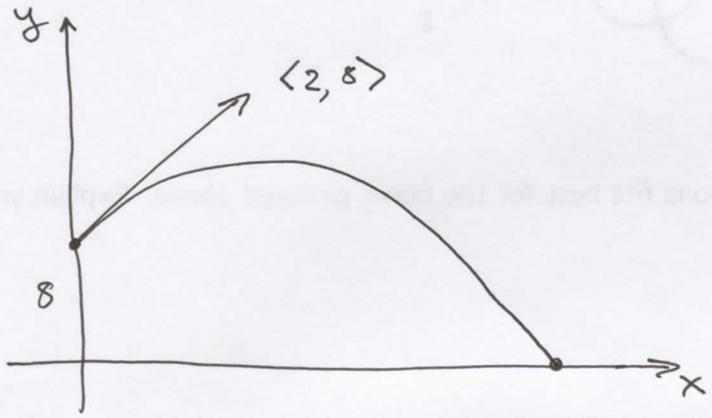
$\boxed{(c)}$ is the only curve that fits.

(15 pts) **7.** An object is hit at 8 feet from the ground with an initial velocity $\mathbf{v}_0 = \langle 2, 8 \rangle \frac{ft}{s}$. Assuming $g = 32\frac{ft}{s^2}$ answer the following questions:

(a) Write down a parametric equation of the trajectory.

(b) Determine the time of flight and range of the object.

(c) What was the maximum hight of the object during its flight?



a) $r(t) = \langle 2t,\ 8 + 8t - 16t^2 \rangle$

b) $y(t) = 0$ when it hits the ground. So,

$8 + 8t - 16t^2 = 0$

$(4t)^2 - 2(4t) + 1 = 9$

$(4t - 1)^2 = 9$

$4t - 1 = \pm 3$

$t = 1$ or $-\frac{1}{2}$ ← unrealistic.

So, $\boxed{t = 1\ s}$

Range $= x(1) = \boxed{2\ ft}$

c) $y'(t) = 0$

$8 - 32t = 0$, $t = \frac{1}{4}\ s$ ← this is when it reaches max height.

$y(\frac{1}{4}) = 8 + 2 - 1 = \boxed{9\ ft}$.

(10 pts) **8.** Find the arc length of the curve given by

$$\mathbf{r}(t) = \left\langle \frac{t^2}{2}, \frac{t^3}{3} \right\rangle,$$

on the range $0 \le t \le 1$.

$$L = \int_a^b |r'(t)|\, dt$$

$$r'(t) = \langle t, t^2 \rangle$$

$$L = \int_0^1 \sqrt{t^2 + t^4}\, dt$$

$$= \int_0^1 \sqrt{1+t^2}\; t\, dt$$

$$= \left[\begin{array}{ll} u = 1+t^2 & u(0) = 1 \\ du = 2t\, dt & u(1) = 2 \end{array} \right]$$

$$= \frac{1}{2} \int_1^2 \sqrt{u}\, du = \frac{1}{2} \frac{u^{3/2}}{3/2} \Big|_1^2$$

$$= \boxed{\frac{1}{3}\left(2^{3/2} - 1\right)}$$

(10 pts) **9.** For the curve given by

$$\mathbf{r}(t) = \langle e^t, \sqrt{2}t, e^{-t} \rangle,$$

compute the curvature at $t = 0$.

$$\kappa = \frac{|r' \times r''|}{|r'|^3}$$

$$r'(t) = \langle e^t, \sqrt{2}, -e^{-t} \rangle$$

$$r''(t) = \langle e^t, 0, e^{-t} \rangle$$

$$r'(0) = \langle 1, \sqrt{2}, -1 \rangle$$

$$r''(0) = \langle 1, 0, 1 \rangle$$

$$r'(0) \times r''(0) = \langle \sqrt{2}, -2, -\sqrt{2} \rangle$$

$$|r' \times r''| = \sqrt{2 + 4 + 2} = \sqrt{8}$$

$$|r'| = \sqrt{1 + 2 + 1} = 2$$

$$\kappa(0) = \frac{\sqrt{8}}{8} = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}}$$

$$\therefore \boxed{\frac{1}{2\sqrt{2}}}.$$

(10 pts) **10.** Find the principal unit normal vector at time t to the curve

$$\mathbf{r}(t) = \langle \cos t, \sqrt{2}\sin t, \cos t \rangle.$$

$r'(t) = \langle -\sin t, \sqrt{2}\cos t, -\sin t \rangle$

$|r'(t)| = \sqrt{\sin^2 t + 2\cos^2 t + \sin^2 t} = \sqrt{2}$

$T(t) = \langle -\frac{1}{\sqrt{2}}\sin t, \cos t, -\frac{1}{\sqrt{2}}\sin t \rangle$

$T'(t) = \langle -\frac{1}{\sqrt{2}}\cos t, -\sin t, -\frac{1}{\sqrt{2}}\cos t \rangle$

$|T'(t)| = \underline{1}$.

So, $\boxed{N(t) = \langle -\frac{1}{\sqrt{2}}\cos t, -\sin t, -\frac{1}{\sqrt{2}}\cos t \rangle}$