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1. **(10pt)**

   (a) Find an equation of the plane containing the points $A(1,-4,-2)$, $B(-1,-3,-5)$ and $C(2,-1,3)$.

   (b) Find an equation of the line, perpendicular to the plane in a), and passing through the point $A(1,4,-2)$.

a) $\vec{AB} = \langle -1-1, -3+4, -5+2\rangle = \langle -2, 1, -3\rangle$

$\vec{AC} = \langle 2-1, -1+4, 3+2\rangle = \langle 1, 3, 5\rangle$

$\vec{n} = \vec{AB}\times\vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 1 & -3 \\ 1 & 3 & 5 \end{vmatrix} = \vec{i}(5+9) - \vec{j}(-10+3) + \vec{k}(-6-1) = \langle 14, 7, -7\rangle$

point $A(1,-4,-2)$

plane: $14(x-1) + 7(y+4) + (-7)(z+2) = 0$

b) $\vec{v} = \vec{n}$

$\vec{r}(t) = \langle 1, 4, -2\rangle + t\langle 14, 7, -7\rangle =$

$\langle 1+14t, 4+7t, -2-7t\rangle$

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2. **(10 pt)** The position function of a particle moving on the plane is given by

$$\mathbf{r}(t) = \langle \cos t \sin t + 2, \cos^2 t \rangle.$$

Find the velocity and acceleration of the particle.

$\vec{v}(t) = \langle -\sin^2 t + \cos^2 t, \; 2\cos t(-\sin t) \rangle =$

$\langle \cos 2t, -\sin 2t \rangle$

$\vec{a}(t) = \langle -2\sin 2t, -2\cos 2t \rangle$

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3. **(15pt)** Are the following pairs of vectors parallel, orthogonal or neither?

(a) $\langle 1, 2, -1\rangle$ and $\langle 4, 1, 6\rangle$.

(b) $\langle 1, 1, 2\rangle$ and $\langle 1, 1, -2\rangle$.

(c) $\langle \sqrt{2}, 1, 0\rangle$ and $\langle -1, -\frac{\sqrt{2}}{2}, 0\rangle$.

a) $\langle 1, 2, -1\rangle \cdot \langle 4, 1, 6\rangle = 4 + 2 - 6 = 0$,
orhogonal

b) $\langle 1, 1, 2\rangle \cdot \langle 1, 1, -2\rangle = 1 + 1 - 4 = -2 \neq 0$,
not orhogonal

$\langle 1, 1, 2\rangle$ is not a scalar multiple of $\langle 1, 1, -2\rangle$, so not parallel.

c) $\langle \sqrt{2}, 1, 0\rangle = -\sqrt{2}\langle -1, -\frac{1}{\sqrt{2}}, 0\rangle$
parallel

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4. **(15 pt)** A particle is moving in space with velocity described by the function

$$\mathbf{v}(t) = \langle te^t + 2, t^2, t \rangle.$$

Time t is measured in seconds. At time $t = 0$ the particle is located at the origin $(0, 0, 0)$.

(a) Compute the position function $\mathbf{r}(t)$.

(b) Write down an integral that will give the total distance traveled by the particle during the first second of its travel. Do not evaluate.

a) $\vec{r}(t) = \int \vec{v}(t)\,dt$

$\int te^t dt = te^t - \int e^t dt = te^t - e^t + C$

$u = t \quad dv = e^t dt$

$du = dt \quad v = e^t$

$\vec{r}(t) = \int \vec{v}(t)dt = \langle te^t - e^t + 2t + C_1, \frac{t^3}{3} + C_2, \frac{t^2}{2} + C_3 \rangle$

$\vec{r}(0) = \langle -e^0 + C_1, C_2, C_3 \rangle = \langle 0, 0, 0 \rangle$

$-1 + C_1 = 0 \qquad C_2 = 0 \qquad C_3 = 0$

$C_1 = 1$

$\vec{r}(t) = \langle te^t - e^t + 2t + 1, \frac{t^3}{3}, \frac{t^2}{2} \rangle$

b) $L = \int_0^1 |\vec{r}\,'(t)|\,dt = \int_0^1 |\vec{v}(t)|\,dt =$

$\int_0^1 \sqrt{(te^t+2)^2 + t^4 + t^2}\,dt$

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5. **(10 pt)** Find an equation of the line of the intersection of the planes

$$Q : x + y = 2 \quad \text{and} \quad R : x - 2y = 5.$$

$\vec{n}_Q = \langle 1, 1, 0 \rangle \qquad \vec{n}_R = \langle 1, -2, 0 \rangle$

$\vec{v} = \vec{n}_Q \times \vec{n}_R = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 0 \\ 1 & -2 & 0 \end{vmatrix} = \vec{k}(-2-1) = -3\vec{k} = \langle 0, 0, -3 \rangle$

point:

$x + y = 2 \Rightarrow x = 2 - y$

$x - 2y = 5$

$2 - y - 2y = 5$

$3y = -3$

$y = -1$

$x = 2 - (-1) = 3$, take $z = 0$,

so $(3, -1, 0)$ is a point in the intersection of two planes.

$\vec{r}(t) = \langle 3, -1, 0 \rangle + t \langle 0, 0, -3 \rangle = \langle 3, -1, -3t \rangle$

6. **(10 pt)** Find the domain of the function

$$f(x, y) = \sqrt{x^2 + y^2 - 1}.$$

Find the equations of the level curves for $z = \sqrt{3}$ and $z = \sqrt{8}$, and sketch the domain and the level curves in the same picture.

$x^2+y^2-1 \geq 0$, $D = \{(x,y) \mid x^2+y^2 \geq 1\}$

level curves:

$\sqrt{3} = \sqrt{x^2+y^2-1}$

$3 = x^2+y^2-1$

$x^2+y^2 = 4$

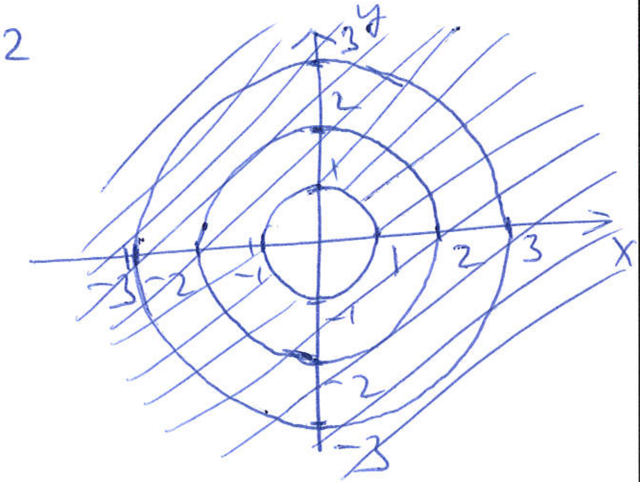
circle of radius 2

$\sqrt{8} = \sqrt{x^2+y^2-1}$

$8 = x^2+y^2-1$

$x^2+y^2 = 9$

circle of radius 3



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7. **(15 pt)** Find all four second order partial derivatives of the function

$$f(x, y) = 3\cos y - ye^{2x} + 5.$$

$f_x = -2ye^{2x}$

$f_y = -3\sin y - e^{2x}$

$f_{xx} = -4ye^{2x}$

$f_{xy} = -2e^{2x}$

$f_{yx} = -2e^{2x}$

$f_{yy} = -3\cos y$

8. **(15 pt)** Consider the function

$$f(x, y) = 3x - y\sqrt{x}$$

(a) Compute the gradient function.

(b) Find the directional derivative of the function at the point $(1, 2)$ in the direction of the vector $\mathbf{u} = \langle -1, 0 \rangle$.

(c) Find the unit vector in the direction of the steepest ascent and the rate of the steepest ascent at $(1, 2)$.

a) $f_x = 3 - \frac{1}{2} y \frac{1}{\sqrt{x}}$

$f_y = -\sqrt{x}$

$\nabla f(x,y) = \langle 3 - \frac{y}{2\sqrt{x}}, -\sqrt{x} \rangle$

b) $\nabla f(1, 2) = \langle 3 - \frac{2}{2}, -1 \rangle = \langle 2, -1 \rangle$

$D_{\vec{u}} f(1,2) = \langle 2, -1 \rangle \cdot \langle -1, 0 \rangle = -2$

c) $\vec{u} = \frac{\nabla f(1,2)}{|\nabla f(1,2)|} = \frac{\langle 2, -1 \rangle}{\sqrt{4+1}} = \langle \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}} \rangle$

rate of max change: $|\nabla f(1,2)| = \sqrt{5}$