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# MATH 210 Exam 1

**February 16, 2017**

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR.

Name: ______________________________

UIN: ______________________________

University Email: ______________________________

Check next to your instructor's name:

| | | |
|---|---|---|
| Lukina | | |
| Abramov | | |
| Heard | | |
| Woolf | | |
| Thulin | | |
| Page | | |
| Skalit | | |
| Kobotis | | |
| Freitag | | |
| Shulman | | |
| Lesieutre | | |

- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your solution will not be read and therefore not graded.
- A solution for one problem may not go on another page.
- Show all your work. Unjustified answers are not correct. Make clear what your final answer is.
- Have your student ID ready to be checked when submitting your exam.

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1. **(10 pt)** Let $\mathbf{u} = \langle 1, 2, 1\rangle$ and $\mathbf{v} = \langle -2, 1, 2\rangle$

   (a) Compute $3\mathbf{u} - 2\mathbf{v}$.

   (b) Compute $\mathbf{u} \cdot \mathbf{v}$.

   (c) Determine if $\mathbf{u}$ and $\mathbf{v}$ are parallel, orthogonal, or neither. Justify your answer.

a) $3\langle 1,2,1\rangle - 2\langle -2,1,2\rangle = \langle 3,6,3\rangle - \langle -4,2,4\rangle =$

$\langle 3+4, 6-2, 3-4\rangle = \langle 7\ 4, -1\rangle$

b) $\vec{u}\cdot\vec{v} = \langle 1,2,1\rangle \cdot \langle -2,1,2\rangle =$

$-2+2+2 = 2$

c) not orthogonal since $\vec{u}\cdot\vec{v} \neq 0$.

not parallel, since $\langle 1,2,1\rangle \neq c\langle -2,1,2\rangle$ for any $c$.

2. **(15pt)** Consider the points $P(1,2,3)$, $Q(4,5,6)$, $R(0,2,0)$.

   (a) Compute the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

   (b) Compute $\overrightarrow{PQ} \times \overrightarrow{PR}$.

   (c) Find the area of the triangle $PQR$.

a) $\overrightarrow{PQ} = \langle 4-1, 5-2, 6-3 \rangle = \langle 3, 3, 3 \rangle$

$\overrightarrow{PR} = \langle 0-1, 2-2, 0-3 \rangle = \langle -1, 0, -3 \rangle$

b) $$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 3 & 3 \\ -1 & 0 & -3 \end{vmatrix} =$$

$$\vec{i} \begin{vmatrix} 3 & 3 \\ 0 & -3 \end{vmatrix} - \vec{j} \begin{vmatrix} 3 & 3 \\ -1 & -3 \end{vmatrix} + \vec{k} \begin{vmatrix} 3 & 3 \\ -1 & 0 \end{vmatrix} =$$

$$\vec{i}(-9) - \vec{j}(-9+3) + \vec{k}(0-(-3)) = \langle -9, 6, 3 \rangle$$

c) area of $PQR = \frac{1}{2} | \overrightarrow{PQ} \times \overrightarrow{PR} |$

$| \overrightarrow{PQ} \times \overrightarrow{PR} | = \sqrt{81+36+9} = \sqrt{126}$

area of $PQR = \frac{1}{2}\sqrt{126}$

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3. **(15pt)** Write down the equations for the following lines:

(a) The line through the point $(0, 1, 1)$ parallel to the line $\mathbf{r}(t) = \langle 2t, 3 - 4t, -1 + 7t \rangle$.

(b) The line through the point $(2, 1, 2)$ perpendicular to the plane $x - 3y + z = 1$.

a) $\vec{V} = \langle 2, -4, 7 \rangle$

$\vec{R}_0 = \langle 0, 1, 1 \rangle$

$\vec{R}(t) = \langle 0, 1, 1 \rangle + t \langle 2, -4, 7 \rangle$

b) normal vector to the plane

$\vec{n} = \langle 1, -3, 1 \rangle$

take $\vec{V} = \vec{n} = \langle 1, -3, 1 \rangle$

$\vec{r}(t) = \langle 2, 1, 2 \rangle + t \langle 1, -3, 1 \rangle$

4. **(15 pt)** A particle moves in space with acceleration given by the vector-valued function

$$\mathbf{a}(t) = \langle t, 2t, \cos t \rangle.$$

Time $t$ is measured in seconds. At time $t = 0$ the particle is located at the origin $(0,0,0)$ and has the velocity $\mathbf{v}_0 = \langle 0, 1, 0 \rangle$.

(a) Compute the velocity function $\mathbf{v}(t)$.

(b) Compute the position function $\mathbf{r}(t)$.

a) $\vec{v}(t) = \int \vec{a}(t)\,dt = \langle \frac{t^2}{2}, 2\frac{t^2}{2}, \sin t \rangle + \langle C_1, C_2, C_3 \rangle$

$\vec{v}(0) = \langle C_1, C_2, C_3 \rangle = \langle 0, 1, 0 \rangle,$

so $C_1 = 0, \quad C_2 = 1, \quad C_3 = 0$

$\vec{v}(t) = \langle \frac{t^2}{2}, t^2 + 1, \sin t \rangle$

b) $\vec{r}(t) = \int \vec{v}(t)\,dt = \langle \frac{1}{2}\frac{t^3}{3} + D_1,$

$\frac{t^3}{3} + t + D_2, -\cos t + D_3 \rangle$

$\vec{r}(0) = \langle D_1, D_2, -1 + D_3 \rangle = \langle 0, 0, 0 \rangle$

$D_1 = 0$

$D_2 = 0$

$-1 + D_3 = 0 \qquad D_3 = 1$

$\vec{r}(t) = \langle \frac{t^3}{6}, \frac{t^3}{3} + t, -\cos t + 1 \rangle$

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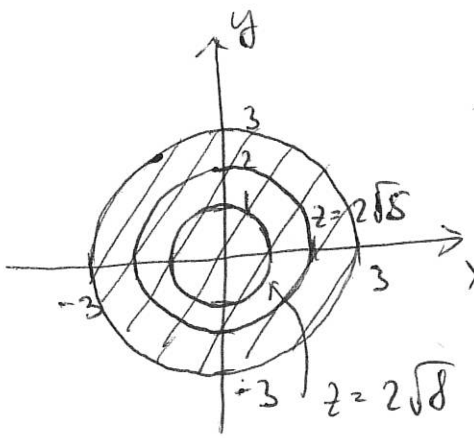
5. **(10 pt)** Let $z = f(x, y) = 2\sqrt{9 - x^2 - y^2}$.

(a) State the domain of $f$. Give a rough sketch of this region in the $xy$ plane; be sure to shade in the area belonging to the domain.

(b) Find the equations of level curves for $z = 2\sqrt{5}$ and for $z = 2\sqrt{8}$. Sketch the level curves on the same picture as the domain of $f$.

a) domain $D = \{ (x, y) \mid 9 - x^2 - y^2 \geq 0 \}$

$x^2 + y^2 \leq 9$ disk of radius 3




b) $2\sqrt{5} = 2\sqrt{9-x^2-y^2}$

$\cancel{4} \cdot 5 = \cancel{4}(9 - x^2 - y^2)$

$x^2 + y^2 = 9 - 5 = 4$

circle of radius 2

$2\sqrt{8} = 2\sqrt{9 - x^2 - y^2}$

$8 = 9 - x^2 - y^2$

$x^2 - y^2 = 9 - 8 = 1$

circle of radius 1

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6. **(10 pt)** Let $\mathbf{r}(t) = \langle t^3 - 9t,\, t^2e^t,\, 6t^2 \rangle$.

(a) Compute $\dfrac{d\mathbf{r}}{dt}$.

(b) Compute the tangent vector for $t = -1$.

$\vec{\ell}(t)$

(c) Compute the equation of the line tangent to the graph of $\mathbf{r}(t)$ for $t = -1$.

a) $\vec{r}\,'(t) = \langle 3t^2 - 9,\, 2te^t + t^2e^t,\, 12t \rangle$

b) $\vec{r}\,'(-1) = \langle 3-9,\, -2e^{-1} + (-1)^2e^{-1},\, -12 \rangle =$

$\langle -6,\, -e^{-1},\, -12 \rangle$

c) $\vec{v} = \vec{r}\,'(-1) = \langle -6,\, -\frac{1}{e},\, -12 \rangle$

$\vec{r}(-1) = \langle (-1)^3 - 9(-1),\, (-1)^2e^{-1},\, 6(-1)^2 \rangle =$

$\langle 8,\, \frac{1}{e},\, 6 \rangle$

$\vec{\ell}(t) = \langle 8,\, \frac{1}{e},\, 6 \rangle + t\langle -6,\, -\frac{1}{e},\, -12 \rangle$

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7. **(10 pt)** Find all four second order partial derivatives of the function

$$f(x, y) = 3xe^y - x^2y + 5.$$

$f_x = 3e^y - 2xy$

$f_y = 3xe^y - x^2$

$f_{xx} = -2y$

$f_{yy} = 3xe^y$

$f_{xy} = 3e^y - 2x$

$f_{yx} = 3e^y - 2x$

8. **(15 pt)** Consider the function $f(x,y) = 3x^2 + 4y^2 - 2$.

(a) Compute the gradient $\nabla f(x,y)$.

(b) Find the directional derivative of $f(x,y)$ at the point $(1,1)$ in the direction of the vector $\mathbf{u} = \langle \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$.

(c) Find the unit vector in the direction of maximum increase and the rate of maximum increase at $(1,1)$.

a) $f_x = 6x \qquad f_y = 8y$

$\nabla f(x,y) = \langle 6x, 8y \rangle$

b) $\nabla f(1,1) = \langle 6, 8 \rangle$

$D_{\vec{u}} \nabla f(1,1) = \langle 6, 8 \rangle \cdot \langle \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle = 3 + 4\sqrt{3}$

c) the direction of max increase is

$\nabla f(1,1) = \langle 6, 8 \rangle$

unit vector in this direction $\dfrac{\langle 6, 8 \rangle}{\sqrt{36+64}} =$

$\langle 0.6, 0.8 \rangle$

the rase of max increase is

$|\nabla f(1,1)| = \sqrt{36+64} = 10$