MATH 210
Sample exam problems for the 1st hour exam
Fall 2009

1. Let $A = (1, -1, 2),\ B = (0, -1, 1),\ C = (2, 1, 1)$.
   (a) Find the vector equation of the plane through A, B, C.
   (b) Find the area of the triangle with these three vertices.
2. Find the vector of length one in the direction of $\overrightarrow{\mathbf{v}} - \overrightarrow{\mathbf{u}}$ where $\overrightarrow{\mathbf{v}} = \langle 7, 5, 3\rangle$ and $\overrightarrow{\mathbf{u}} = \langle 4, 5, 7\rangle$.
3. Let $\overrightarrow{\mathbf{r}}(t) = \big\langle 3t - 1, e^t, \cos(t)\big\rangle$.
   (a) Find the unit tangent vector $\overrightarrow{\mathbf{T}}$ to the path $\overrightarrow{\mathbf{r}}(t)$ at $t = 0$.
   (b) Find the speed, $\big|\big|\overrightarrow{\mathbf{r}}'(t)\big|\big|$ at $t = 0$.
4. Given a point $P = (0, 1, 2)$ and the vectors $\overrightarrow{\mathbf{u}} = \langle 1, 0, 1\rangle$ and $\overrightarrow{\mathbf{v}} = \langle 2, 3, 0\rangle$, find
   (a) an equation for the plane that contains P and whose normal vector is perpendicular to the two vectors $\overrightarrow{\mathbf{u}}$ and $\overrightarrow{\mathbf{v}}$,
   (b) a set of parametric equations of the line through P and in the direction of $\overrightarrow{\mathbf{v}}$.
5. Find the speed and arclength of the path $\overrightarrow{\mathbf{r}}(t) = \langle 3\cos t, 4\cos t, 5\sin t\rangle$ where $0 \le t \le 2$.
6. Find the curvature at $t = 0$ for the curve $\overrightarrow{\mathbf{r}}(t) = e^t\,\hat{\mathbf{i}} + t^2\,\hat{\mathbf{j}} + t\,\hat{\mathbf{k}}$.
7. Let $\overrightarrow{\mathbf{r}}(t) = \langle t, \cos t, \sin t\rangle$.
   (a) Find the velocity vector, $\overrightarrow{\mathbf{r}}'(t)$.
   (b) Find the acceleration vector, $\overrightarrow{\mathbf{r}}''(t)$.
   (c) Find the component of acceleration in the direction of the velocity when $t = 0$.
8. Let $f(x, y) = \frac{1}{2}x^2 - y$. Sketch the three level curves on which $f(x, y) = -1$ or 0 or 1 in the square $-2 \le x \le 2, -2 \le y \le 2$.
9. Find the partial derivatives
$$\frac{\partial f}{\partial x}, \qquad \frac{\partial f}{\partial y}, \qquad \text{and} \qquad \frac{\partial^2 f}{\partial x \partial y}$$
   for the function $f(x, y) = 2x + 3xy - 5y^2$.
10. Find the partial derivatives
$$\frac{\partial^2 f}{\partial x^2} \qquad \text{and} \qquad \frac{\partial^2 f}{\partial y^2}$$
   for the function $f(x, y) = e^{2x}\cos(2y)$.