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Math 210 Hour Exam One

1a. Consider the curve $\vec{\mathbf{r}}(t) = (t, t^2, \frac{2}{3}t^3)$.

(1) Find the arc length of $\vec{\mathbf{r}}(t)$ from $t = 0$ to $t = 1$;
(2) Find the curvature at $t = 1$.

1b. A particle moves along the space curve $\vec{\mathbf{r}}(t) = (t\cos(t), t\sin(t), t)$.

(1) Find the velocity, acceleration, and speed as functions of time.
(2) Find the unit tangent and unit normal vectors at $t = 0$.

2a. Let $f(x, y) = x\sqrt{y} + y$.

(1) Find f_x and f_y at $(2, 4)$.
(2) Write the equation of the tangent plane to the graph of f at the point $(2, 4)$.

2b. Let $f(x, y) = \sqrt{x^2 + 3y^2}$. Note that $f(1, 4) = 7$.

(1) Find f_x and f_y at $(1, 4)$.
(2) Write the equation of the tangent plane to the graph of f at the point $(1, 4)$.

3a. Compute $\frac{dw}{dt}$ for $w = e^{-x}\sin(x + y)$, where $x = t^2$ and $y = 1 - t$.

3b. Compute $\frac{dw}{dt}$ for $w = e^{y-x}\sin y$, where $x = t^2$ and $y = 1 - t$.

4a. Let $f(x, y, z) = x^2 + yz$.

(1) Compute the gradient of f.
(2) Find the derivative of f at $(1, 1, -3)$ in the direction $\vec{\mathbf{u}} = \frac{1}{3}(2\vec{\mathbf{i}} + \vec{\mathbf{j}} + 2\vec{\mathbf{k}})$.
(3) In what direction is f increasing most rapidly at $(1, 1, -3)$?

4b. Let $f(x, y) = x^2 y$.

(1) Compute the gradient of f.
(2) Find the derivative of f at $(1, 2)$ in the direction $\vec{\mathbf{u}} = \langle \frac{4}{5}, -\frac{3}{5} \rangle$.
(3) In what direction is f increasing most rapidly at $(1, 2)$?

5a. Consider three points $A = (0, 0, 0)$, $B = (3, 1, -1)$, $C = (1, 1, 1)$.

(1) Find the equation for the plane which contains the points A, B, C.
(2) What is the area of the triangle ABC?

5b. Consider the plane $z + y + 2z = 4$ and the point $P = (1, 0, 1)$.

(1) Find the line through P that is perpendicular to the plane.
(2) At what point does the line from (1) intersect the plane.
(3) Find the distance between P and the plane.