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MATH 210 Exam 2

October 29, 2015

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR.

Name: ____________________

UIN: ____________________

University Email: ____________________

Check next to your instructor's name:

Lukina	11am	
Lukina	1pm	
Kobotis	8am	
Greenblatt	10am	
Greenblatt	1pm	
Goldbring	11am	
Hong	10am	
Hong	12pm	
Dumas	12pm	
Dai	2pm	
Heard	9am	
Wang	2pm	
Torres	9am	

- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your exam will not be read and therefore not graded.
- A solution for one problem may not go on another page.
- Show all your work. Unjustified answers are not correct. Make clear to the grader what your final answer is.
- Have your student ID ready to be checked when submitting your exam.

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1. **(15pt)** Consider the function
$$f(x, y) = \sin(2x - 2y).$$

(a) Find the gradient of the function.

(b) Compute the directional derivative of the function at the point $P(\frac{\pi}{2}, \frac{\pi}{6})$ in the direction of the vector $\vec{u} = \langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \rangle$.

(c) Find the unit vector in the direction of the steepest ascent at $P(\frac{\pi}{2}, \frac{\pi}{6})$.

a) $\nabla f(x,y) = \langle 2\cos(2x-2y), -2\cos(2x-2y) \rangle$

b) $\nabla f(\frac{\pi}{2}, \frac{\pi}{6}) = \langle 2\cos(\pi - \frac{\pi}{3}), -2(\pi - \frac{\pi}{3}) \rangle =$

$\langle 2\cos\frac{2\pi}{3}, -2\cos\frac{2\pi}{3} \rangle = \langle -1, 1 \rangle$

$D_{\vec{u}} f(\frac{\pi}{2}, \frac{\pi}{6}) = \langle -1, 1 \rangle \cdot \langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \rangle =$

$-\frac{\sqrt{3}}{2} - \frac{1}{2}$

c) $\nabla f(\frac{\pi}{2}, \frac{\pi}{6}) = \langle -1, 1 \rangle$

$$\frac{\nabla f(\frac{\pi}{2}, \frac{\pi}{6})}{|\nabla f(\frac{\pi}{2}, \frac{\pi}{6})|} = \frac{\langle -1, 1 \rangle}{\sqrt{1+1}} = \langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$$

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2. **(10pt)** Find the equation of the tangent plane to the surface

$$2xy + ze^y = 0$$

at the point $(e, 1, -2)$.

$\nabla F(x,y,z) = \langle 2y,\ 2x + \cancel{z}e^y,\ e^y \rangle$

$\nabla F(e,1,-2) = \langle 2,\ 2e - 2e,\ e \rangle = \langle 2, 0, e \rangle$

$2(x-e) + 0(y-1) + e(z+2) = 0$

$2(x-e) + e(z+2) = 0.$

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3. **(10pt)** Use the linear approximation to the function

$$f(x, y) = x^3 - 2y^2$$

at $(1, 1)$ to estimate $f(1.05, 0.9)$.

$\nabla f(x,y) = \langle 3x^2, -4y \rangle$

$\nabla f(1,1) = \langle 3, -4 \rangle$

$f(1,1) = 1 - 2 = -1$

$L(x,y) = 3(x-1) - 4(y-1) - 1$

$L(1.05, 0.9) = 3(1.05-1) - 4(0.9-1) - 1 =$

$0.15 + 0.4 - 1 = -0.45$

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4. **(20 pt)** Consider the function

$$f(x, y) = x^3 + 3xy + y^3$$

(a) Find the critical points of the function.

(b) Use the Second Derivative Test to classify each critical point as a local maximum, local minimum, or a saddle point.

a) $f_x = 3x^2 + 3y = 0$

$f_y = 3x + 3y^2 = 0 \qquad x = -y^2$

$3y^4 + 3y = 0$

$3y(y^3+1) = 0$, so $y = 0$ or $y^3 = -1$

$y = -1$.

then $x = 0$, $x = -1$,

crit. points $(0,0)$, $(-1,-1)$.

b) $f_{xx} = 6x$, $f_{yy} = 6y$, $f_{xy} = 3$

$D(x,y) = 36xy - 9$

$D(0,0) = -9 < 0$ saddle point

$D(-1,-1) = 36 - 9 > 0$

$f_{xx}(-1,-1) = 6(-1) = -6 < 0$

local max

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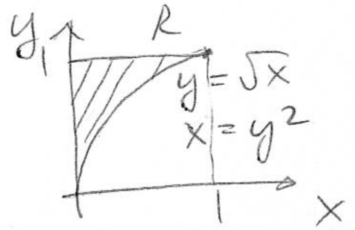
5. **(10 pt)** For the double integral

$$\int_0^1 \int_{\sqrt{x}}^1 \sqrt{2+y^3}\, dydx$$

(a) Sketch the region of integration.

(b) Change the order of integration.

(c) Evaluate the integral.

a) 

$R = \{(x,y) \mid 0 \le x \le 1,\ \sqrt{x} \le y \le 1\}$

b) $R = \{(x,y) \mid 0 \le y \le 1,\ 0 \le x \le y^2\}$

$$\int_0^1 \int_0^{y^2} \sqrt{2+y^3}\, dx\, dy$$

c) $$\int_0^1 \int_0^{y^2} \sqrt{2+y^3}\, dx\, dy = \int_0^1 \sqrt{2+y^3}\; x \Big|_0^{y^2} dy =$$

$$\int_0^1 y^2 \sqrt{2+y^3}\, dy$$

substitution: $u = 2+y^3$ if $y=0$, $u=2$

$du = 3y^2 dy$ $y=1$, $u=3$

$$\int_0^1 y^2\sqrt{2+y^3}\, dy = \frac{1}{3}\int_2^3 u^{1/2}\, du = \frac{1}{3}\cdot\frac{2}{3} u^{3/2}\Big|_2^3 =$$

$$\frac{2}{9}\left(3^{3/2} - 2^{3/2}\right)$$

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6. **(15 pt)** For the function
$$f(x,y) = x^2 + 3y^2 - y$$
subject to the constraint
$$x^2 + 2y^2 = 2,$$
use the method of Lagrange multipliers to find the maximum and the minimum values of the function, and all points where these values are achieved.

$\nabla f(x,y) = \langle 2x, 6y-1 \rangle$

$g(x,y) = x^2 + 2y^2 - 2$, $\nabla g(x,y) = \langle 2x, 4y \rangle$

$2x = \lambda\, 2x \Rightarrow \lambda = 1$ or $x = 0$

$6y - 1 = \lambda\, 4y$ if $\lambda = 1$: $6y - 1 = 4y$

$x^2 + 2y^2 = 2$ $2y = 1$

$y = \frac{1}{2}$

$x = \pm\sqrt{2 - 2 \cdot \frac{1}{4}} = \pm\sqrt{\frac{3}{2}}$,

points $(\pm\frac{3}{2}, \frac{1}{2})$

if $x = 0$, $y^2 = 1$, $y = \pm 1$,

points $(0, \pm 1)$

$f(0,1) = 3 - 1 = 2$

$f(0,-1) = 3 + 1 = 4$

$f(\sqrt{\frac{3}{2}}, \frac{1}{2}) = \frac{3}{2} + \frac{3}{4} - \frac{1}{2} = \frac{7}{4}$

$f(-\sqrt{\frac{3}{2}}, \frac{1}{2}) = \frac{3}{2} + \frac{3}{4} - \frac{1}{2} = \frac{7}{4}$

min: $f(\pm\sqrt{\frac{3}{2}}, \frac{1}{2}) = \frac{7}{4}$

max: $f(0,-1) = 4$

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7. **(13 pt)** Consider the integral

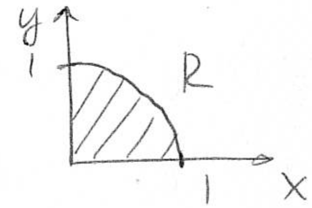
$$\iint_R (x+y)dA$$

where R is the region in the first quadrant bounded by the parabola $y = 1-x^2$ and by the coordinate axes.

(a) Sketch the region R.

(b) Evaluate the integral.

a)



b)

$$\iint_R (x+y)dA = \int_0^1 \int_0^{1-x^2} (x+y)\,dy\,dx =$$

$$\int_0^1 \left(xy + \frac{y^2}{2}\right)\Big|_0^{1-x^2} dx = \int_0^1 \left(x(1-x^2) + \frac{(1-x^2)^2}{2}\right)dx =$$

$$\int_0^1 \left(x - x^3 + \frac{1}{2}(1-2x^2+x^4)\right)dx =$$

$$\int_0^1 \left(\frac{1}{2} + x - x^2 - x^3 + \frac{x^4}{4}\right)dx =$$

$$\left(\frac{1}{2}x + \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + \frac{1}{4}\frac{x^5}{5}\right)\Big|_0^1 =$$

$$\frac{1}{2} + \frac{1}{2} - \frac{1}{3} - \frac{1}{4} + \frac{1}{20}$$

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8. **(7 pt)** Find the volume of the solid bounded from above by the paraboloid $z = x^2 + y^2$, from below by the xy-plane, and on the sides by the cylinder $x^2 + y^2 = 4$.

$R = \{(x,y) \mid x^2+y^2 \leq 4\}$

polar coordinates:

$R = \{(r,\theta) \mid 0 \leq \theta \leq 2\pi,\ 0 \leq r \leq 2\}$

$$V = \int_0^{2\pi} \int_0^2 (r^2 - 0)\, r\,dr\,d\theta =$$

$$\int_0^{2\pi} \frac{r^4}{4}\Big|_0^2 d\theta = \frac{16}{4} \int_0^{2\pi} d\theta =$$

$$4\theta \Big|_0^{2\pi} = 8\pi$$