DO NOT WRITE ABOVE THIS LINE!!

MATH 210 Exam 2

March 15, 2016

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR.

Name: ________________________________

UIN: ________________________________

University Email: ________________________________

Check next to your instructor's name:

Lukina	12pm	
Lukina	2pm	
Kobotis	8am	
Whyte	10am	
Lenz	11am	
Abernethy	12pm	
Heard	9am	
Wang	2pm	
Kauffman	3pm	

- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your solution will not be read and therefore not graded.
- A solution for one problem may not go on another page.
- Show all your work. Unjustified answers are not correct. Make clear what your final answer is.
- Have your student ID ready to be checked when submitting your exam.

DO NOT WRITE ABOVE THIS LINE!!

1. **(15pt)** You are standing on the side of a mountain whose shape is described by the graph of the function $f(x, y) = -2x^2 - 3y^2 + xy + 2000$. You are currently located at the point $(3, 2, 1976)$.

   (a) Find the gradient function $\nabla f(x, y)$, and the gradient vector at $(3, 2)$.

   (b) You decide you want to walk up the mountain along the steepest path. What is the direction of the steepest ascent and the rate of the steepest ascent at the point $(3, 2, 1976)$?

a) $\nabla f(x,y) = \langle -4x + y, -6y + x \rangle$

$\nabla f(3,2) = \langle -4 \cdot 3 + 2, -6 \cdot 2 + 3 \rangle = \langle -10, -9 \rangle$

b) $\nabla f(3,2)$ gives the direction of the steepest ascent at $(3,2)$, $\nabla f(3,2) = \langle -10, -9 \rangle$

rate : $|\nabla f(3,2)| = \sqrt{100 + 81} = \sqrt{181}$

DO NOT WRITE ABOVE THIS LINE!!

2. **(15 pt)** Consider the surface described by the equation

$$6z + \sin(xy) - 1 = 0.$$

(a) Verify that the point $P(\pi, 3, \frac{1}{6})$ is on the surface.

(b) Find the tangent plane to the surface at the point $P(\pi, 3, \frac{1}{6})$.

a) $6 \cdot \frac{1}{6} + \sin 3\pi - 1 = 1 + 0 - 1 = 0$

b) $F(x, y, z) = 6z + \sin(xy) - 1$

$\nabla F(x, y, z) = \langle y \cos(xy), x \cos(xy), 6 \rangle$

$\nabla F(\pi, 3, \frac{1}{6}) = \langle 3 \cos 3\pi, \pi \cos 3\pi, 6 \rangle = \langle -3, -\pi, 6 \rangle$

$-3(x - \pi) - \pi(y - 3) + 6(z - \frac{1}{6}) = 0$

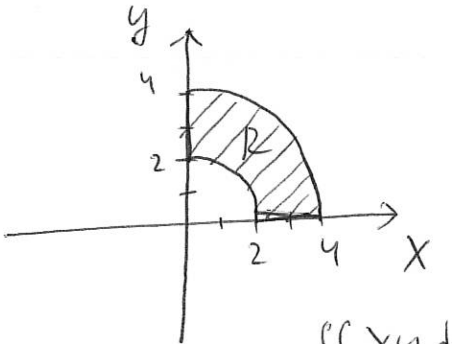
$-3x - \pi y + 6z = -3\pi - 3\pi + 1 = 1 - 6\pi$

$-3x - \pi y + 6z = 1 - 6\pi$

3. **(15 pt)** Evaluate the integral

$$\iint_R xy \, dA,$$

where R is the portion of the region between the circles of radius 2 and 4, centered at the origin, which lies in the first quadrant (Hint: convert the integral to polar coordinates first).



$$R = \{ (r, \theta) \mid 0 \le \theta \le \tfrac{\pi}{2}, \ 2 \le r \le 4 \}$$

$$x = r\cos\theta, \quad y = r\sin\theta,$$

$$\iint_R xy\, dA = \int_0^{\frac{\pi}{2}} \int_2^4 r^2 \cos\theta \sin\theta \, r \, dr \, d\theta =$$

$$\int_0^{\frac{\pi}{2}} \cos\theta \sin\theta \int_2^4 r^3 \, dr \, d\theta = \int_0^{\frac{\pi}{2}} \cos\theta \sin\theta \left(\frac{r^4}{4} \Big|_2^4 \right) d\theta =$$

$$\int_0^{\frac{\pi}{2}} \cos\theta \sin\theta \, (64 - 4) \, d\theta = \frac{60}{2} \int_0^{\frac{\pi}{2}} \sin 2\theta \, d\theta =$$

$$\frac{1}{2} \cdot 30 \left(-\cos 2\theta \right) \Big|_0^{\frac{\pi}{2}} = 15 \left(-\cos\pi + \cos 0 \right) =$$

$$15\,(2) = 30$$

4. **(20 pt)** Find all critical points of

$$f(x,y) = x^2 + y^2 + x^2y + 4$$

and classify them as local maxima, local minima or saddle points.

$\nabla f(x,y) = \langle 2x + 2xy,\ 2y + x^2 \rangle$

$2x + 2xy = 0 \quad \Rightarrow \quad 2x(y+1) = 0, \quad x = 0$ or $y = -1$

$2y + x^2 = 0$

if $x = 0$, then $2y = 0$, $(0,0)$ crit. point

if $y = -1$, then $-2 + x^2 = 0$, $x = \pm\sqrt{2}$, $(-\sqrt{2}, -1)$, $(\sqrt{2}, -1)$ crit. points

$f_{xx} = 2 + 2y \quad f_{yy} = 2, \quad f_{xy} = 2x$

$D(x,y) = 2(2+2y) - 4x^2$

$D(0,0) = 4 > 0 \qquad f_{xx}(0,0) = 2 > 0$, loc. min

$D(-\sqrt{2}, -1) = -4 \cdot 2 < 0$ saddle point

$D(\sqrt{2}, -1) = -4 \cdot 2 < 0$ saddle point

DO NOT WRITE ABOVE THIS LINE!!

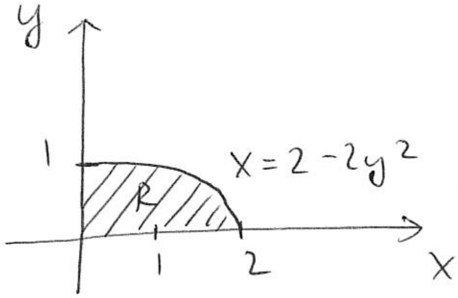
5. **(20 pt)** Consider the integral

$$\iint_R (1 - x - y^2)\,dA$$

where R is the region in the first quadrant bounded by the parabola $x = 2 - 2y^2$ and by the coordinate axes.

(a) Sketch the region R.

(b) Evaluate the integral.



$R = \{(x,y) \mid 0 \le y \le 1,$
$0 \le x \le 2 - 2y^2$

$\iint_R (1-x-y^2)\,dA = \int_0^1 \int_0^{2-2y^2} (1-x-y^2)\,dx\,dy =$

$\int_0^1 \left(x - \frac{x^2}{2} - y^2 x \right)\Big|_0^{2-2y^2} dy = \int_0^1 2 - 2y^2 - \frac{1}{2}(4 - 8y^2 +$
$4y^4) - y^2(2-2y^2)\,dy = \int_0^1 (\cancel{2} - \cancel{2y^2} - \cancel{2} + \cancel{4y^2} - \cancel{2y^4} -$
$\cancel{2y^2} + \cancel{2y^4})\,dy = \int_0^1 0\,dy = 0.$

6. **(15 pt)** Find the absolute maximum and the absolute minimum of the function

$$f(x, y) = x^2 - 2y^2 + 4$$

on the region $R = \{(x, y) \mid x^2 + y^2 \leq 4\}$, and also find all points where these values are achieved.

$f_x = 2x = 0$ (0,0) crit. point

$f_y = -4y = 0$ $f(0,0) = 4$

$g(x,y) = x^2 + y^2 - 4 = 0$

$\nabla g(x,y) = \langle 2x, 2y \rangle$

$2x = \lambda 2x \Rightarrow x = 0$ or $\lambda = 1$

$-4y = \lambda 2y$ $y^2 = 4$ $-4y = 2y$, so $y = 0$

$x^2 + y^2 = 4$ $y = \pm 2$ $x^2 = 4$

points $(0,-2)$, $(0,2)$ $x = \pm 2$

points $(-2,0)$, $(2,0)$

$f(0,-2) = -2 \cdot 4 + 4 = -4$

$f(0,2) = -2 \cdot 4 + 4 = -4$

$f(2,0) = 4 + 4 = 8$

$f(-2,0) = 4 + 4 = 8$

$f(x,y)$ has ~~the~~ absolute max $f(2,0) = f(-2,0) = 8$ on $x^2 + y^2 \leq 4$

$f(x,y)$ has the absolute min $f(0,-2) = f(0,2) = -4$ on $x^2 + y^2 \leq 4$.