MATH 220 Final Exam Summer 2005

(1) Find the general solutions to

(a) $y' + \sin(x)y = \sin(x)$

(b) $y' = x^2y^2$.

(2) Find the general solution to

$$y' = y - xy^2.$$

Hint: Let $y = 1/v$ and derive an equation for $v(x)$.

(3) Use the Euler method with step size $h = 0.5$ to obtain an approximation to $y(1)$ for the problem

$$y' = \frac{1}{1+xy}, \qquad y(0) = 1.$$

(4) Solve the following ODEs

(a) $y'' + 4y' + 3y = e^{3x} + e^{-3x}$

(b) $y'' + 2y' = 1 + \sin(x)$.

(5) Find the general solution to the ODE

$$y'' + xy' - y = x^2.$$

Hint: Find a solution to the homogeneous problem as a polynomial of low degree. Then find a second solution using reduction of order. You can find a particular solution as another polynomial of low degree.

(6) Consider the following system of ODEs for $x(t)$ and $y(t)$

$$x' = y + t, \qquad y' = -x + 1.$$

(a) Find the general solution.

(b) Solve the initial value problem with $x(0) = 0$ and $y(0) = 1$.

(7) Use Laplace transforms to solve for $y(t)$

$$y'' + 3y' + 2y = \delta(t-2), \qquad y(0) = 1, \qquad y'(0) = 0.$$

(8) Consider the Airy equation

$$y''(x) = x\, y(x).$$

(a) For any solution, what is $y''(0)$?

(b) Find the first three nonzero terms in the Taylor expansion about $x = 0$ for the solution that satisfies $y(0) = 1$ and $y'(0) = 0$.

(c) Repeat (b) for the initial conditions $y(0) = 0$ and $y'(0) = 1$.

(9) Solve the heat equation

$$u_t = u_{xx}, \quad 0 < x < 1, \quad t > 0$$

$$u(0,t) = u(1,t) = 0$$

$$u(x,0) = 3\sin(2\pi x).$$

(10) Consider the periodic function

$$f(x) = x, \quad -1 < x < 1; \quad f(x+2) = f(x).$$

(a) Sketch $f(x)$ over the range $-3 < x < 3$.

(b) What is the average value of $f(x)$?

(c) Find the Fourier series for $f(x)$.