

(1) Consider the forced oscillator

$$x''(t) + 2x'(t) + x(t) = A \sin(t)$$

(a) Find the general solution.

(b) If  $A = 0$  is the free motion underdamped, overdamped, or critically damped ?

(c) For  $A > 0$  is the motion oscillatory for large  $t$  ? If yes, what are the amplitude and frequency of this oscillation ?

(2) Consider the following system of ODEs for  $x(t)$  and  $y(t)$

$$x' = 2x + y, \quad y' = x + 2y.$$

(a) Find the general solution.

(b) Solve the initial value problem with  $x(0) = 1$  and  $y(0) = 0$ .

(3) Use Laplace transforms to solve for  $y(t)$

$$y'' + 3y' + 2y = 1 + e^{-t}, \quad y(0) = 0, \quad y'(0) = 1.$$

(4) Use Laplace transforms to solve for  $y(t)$

$$y'' + y = u(t - 1), \quad y(0) = 1, \quad y'(0) = 0.$$

Here  $u$  is the Heaviside step function.

(5) Consider the first order ODE

$$y'(x) = (1 + x)y(x), \quad y(0) = 1.$$

Find the first four (4) nonzero terms in the Taylor expansion of  $y(x)$  about  $x = 0$ .