1. Consider

\[ \int_{-\pi/2}^{\pi/2} e^{\lambda \cos t} dt \]

Find the leading term as (a) \( \lambda \to +\infty \) \hspace{1cm} (b) \( \lambda \to -\infty \).

2. Consider

\[ \int_0^1 t^N e^{i \lambda t^3} dt \]

Find the leading term as \( \lambda \to +\infty \) if (a) \( N = 0 \), \hspace{1cm} (b) \( N = 2 \), \hspace{1cm} (c) \( N = 3 \).

3. Consider the ODE

\[ y'' - (1 + x)y' + xy = 0. \]

Find the leading behavior of all solutions as \( x \to +\infty \).

4. Consider \( F(z) = z^3/3 - z \)

(a) Locate all saddle points of \( F(z) \).

(b) Give the steepest descent (SD) and steepest ascent (SA) directions at each saddle.

(c) Calculate and sketch the SD contour through each saddle in (a).

5. Use the WKB method to find the general solution of:

\[ \varepsilon^2 y'' + \varepsilon^2 p(x)y' - (x^2 + 1)y = 0, \hspace{1cm} \varepsilon \to 0. \]

6. Consider

\[ y'' + y' + 9y^2 + 4\lambda^2 = 1 \]

Describe the steady states, locate and classify the bifurcation points, determine the stability of each branch, and sketch the bifurcation diagram.

7. Compute a leading order composite or uniform approximation as \( \varepsilon \to 0^+ \):

\[ \varepsilon y'' + 2y' + y^3 = 0; \hspace{1cm} y(0) = 0, \hspace{1cm} y(1) = 1/2. \]

8. Use the multiple scales method to obtain an asymptotic approximation \( (\varepsilon \to 0^+) \) to \( y = y(t) \) that is valid for all times \( t = O(\varepsilon^{-1}) \)

\[ y'' + \varepsilon (y')^3 + y = 0; \hspace{1cm} y(0) = 0; \hspace{1cm} y'(0) = 1 \]