Qualifiers Questions in Algorithms

MCS 501

Spring 2008

**Question 1** Suppose we are given $n$ points which lie in the disc $x^2 + y^2 \leq 1$. We know that these points were chosen at random with uniform probability. (That is the probability of finding a point in any region of the circle is proportional to the area of that region.) We need to sort each point based on its length. If a point is given by $(x_1, y_1)$ its length is $\sqrt{x_1^2 + y_1^2}$. Design an efficient sorting algorithm for this problem. Please do not write a code. Just give a clear and precise description or pseudocode. (First say which sorting algorithm you will use and why. Next describe how you will design it.)

**Question 2** Prove that to merge two sorted lists of length $n$ we must make $2n - 1$ comparisons in the worst case.

*Hint*: First show that if two consecutive numbers are in different lists, then they must be compared.

**Question 3** Given a function which produces a random integer (with uniform probability) in the range 1 to 5, write a function which produces a random integer in the range 1 to 7 (with uniform probability).

**Question 4** Suppose you have an oracle to solve 3-SAT in polynomial time. How can you use that oracle to check if a graph can be 3-coloured in polynomial time?