1. Find the Galois group of $x^5 - 7$ over $\mathbb{Q}$.

2. Let $G$ be a group of order 77. Show that $G$ is cyclic.

3. Let $R$ be a ring with 1, and let $u, v \in R$ satisfy $uv = 1$ (so $v$ is a right inverse of $u$). Prove that the following are equivalent:
   (a) There is a nonzero element $w$ in $R$, such that $uw = 0$;
   (b) $u$ is not a unit;
   (c) $u$ has more than one right inverse.

4. Let $G$ be a group. Suppose $g \in G$. Denote by $C_g$ the automorphism of $G$ induced by conjugation by $g$, so $C_g(x) = gxg^{-1}$, for each $x \in G$.
   (a) Consider the map $\phi : G \rightarrow \text{Aut}(G)$ defined by $\phi(g) = C_g$. Show that $\phi$ is a group homomorphism.
   (b) Show that $\text{Ker}(\phi) = Z(G)$, the center of $G$. Suppose that $\alpha \in \text{Aut}(G)$. Show that $\alpha C_g \alpha^{-1} = C_{\alpha(g)}$. Show that the image of $\phi$ is a normal subgroup of $\text{Aut}(G)$.
   (c) Show that if $Z(G)$ is trivial, then $Z(\text{Aut}(G))$ is also trivial.

5. Let $G$ be a finite group. Suppose that $K$ is a normal subgroup of $G$ and $P$ is a $p$–Sylow subgroup of $K$. Show that $G = KN_G(P)$.

6. Show that $\mathbb{Q}$ is not a finitely generated module over $\mathbb{Z}$.

7. Let $A$ be a nonzero finite abelian group.
   (a) Prove that $A$ is not a projective $\mathbb{Z}$-module.
   (b) Prove that $A$ is not an injective $\mathbb{Z}$-module.

8. Let $p$ be a prime number. Suppose $L$ is a finite field with $p^{10}$ elements. How many subfields does $L$ have?