1. Let $E$ be a measurable set of $\mathbb{R}$ and let $m(E)$ refer to Lebesgue measure of $E$. Suppose $m(E) < \infty$. For $t \in \mathbb{R}$ let $E + t = \{x + t : x \in E\}$. Show that
$$\lim_{t \to 0} m(E \Delta (E + t)) = 0.$$

2. Show that an absolutely continuous function defined on a bounded interval of the reals has bounded variation. Show the statement is false if the interval is unbounded.

3. Suppose $f_k \to f$ in $L^p(m)$, $p > 1$, where $m$ is Lebesgue measure on $\mathbb{R}$. Show that
$$\int f_k g \to \int fg$$
for all $g \in L^q$ where $\frac{1}{p} + \frac{1}{q} = 1$. Show the converse is false.

4. Construct a function which is in $L^1(-\infty, \infty)$ but is not in $L^2(a, b)$ for any $a, b$. Assume the measure is Lebesgue. Hint: Start with $g(x) = x^{-1/2}$ on $(0, 1)$ and 0 elsewhere and let $f(x) = \sum a_k g(x - r_k)$ where $r_k$ are the rationals and $a_k$ are constants to be chosen.

5. Given an integrable function $f$ on a measure space $(X, m)$ define $F : [0, \infty) \to [0, \infty)$ by $F(t) = m\{x \in X : |f(x)| \geq t\}$. Show that
$$\int_X |f(x)|\, dm(x) = \int_0^{\infty} F(t)\, dt.$$

6. Show that if $g \in L^1(X, m)$ then for every $\epsilon > 0$ there exists a $\delta > 0$ so that if $E \subset X$ is measurable and $m(E) < \delta$ then $\int_E |g|\, dm < \epsilon$.

7. Show that if $f \in L^1(\mathbb{R})$ satisfies $\int_0^b f(x)\, dx = 0$ whenever $b - a$ is rational, then $f(x) = 0$ a.e.

8. a) Show that $|\sin(x)/x| \notin L^1(0, \infty)$.
   b) Show that $\int_0^\infty \sin(x)/x = \pi/2$ by applying iterated integration to the function $e^{-xy}\sin(x)$.