Algebraic Topology Preliminary Exam
May 12, 2000

Do any five of the following eight problems.

1. Show that $H_j(X, A) \cong H_j(X \cup CA, P)$ where $CA = A \times I/(a, 0) ~ v$ is the unreduced cone on $A$, $(a, 1) \in CA$ identified with $a \in A \subset X$, and $P = \{ v \}$.

2. Let $X$ and $Y$ be disjoint, closed subsets of $S^n$, $n \geq 2$. Show that $S^n - X$ and $S^n - Y$ are connected if and only if $S^n - X \cup Y$ is connected.

3. Let $0 \to C_* \to D_* \to E_* \to 0$ be an exact sequence of chain complexes. Define the connecting homomorphism

$$H_i(E) \xrightarrow{\partial} H_{i-1}(C)$$

on cycles in $E_i$ and show that it is well-defined on homology classes. Show that

$$H_i(D) \xrightarrow{i_*} H_i(E) \xrightarrow{\partial} H_{i-1}(C)$$

is exact.

4. Calculate $H_*(RP^3 \times RP^2; Z)$.

5. If the closed 4-dimensional manifold $M$ has a nonvanishing vector field, show that $M$ is not simply connected.

6. A certain 3-manifold $L$ has a cellular chain complex $\{ C_*, \partial_* \}$ with $C_i = Z$ for $0 \leq i \leq 3$, $\partial_3 = 0$, $\partial_2 : C_2 \to C_1$ is multiplication by 3, and $\partial_1 = 0$. Calculate the integral homology of $L$ and the cohomology with $Z/3Z$ coefficients. Which cup products in $H^*(L; Z/3Z)$ are nonzero?

7. Let $\tilde{X}$ be the universal cover of $RP^2 \vee S^2$. Calculate $H_2(\tilde{X})$.

8. Let $f : S^8 \times RP^2 \to S^8 \times RP^2$ with $f$ homotopic to the identity map. Show that $f$ must have a fixed point.