1. Let $M$ be a structure in a language with one binary relation $E$. Suppose that on $M$, $E$ is an equivalence relation with infinitely many classes with 2 elements and one class with one element. Let $T$ be the theory of $M$.

Does $T$ admit elimination of quantifiers in the language $L$? If not, describe a definable expansion of $T$ which is.

Is $T$ model complete?

What is a set of axioms for $T$? Is $T$ categorical in any infinite cardinalities?

Sketch arguments to justify each of your responses.

2. Write one or two paragraphs to explain the meaning and significance of the following statements to a research mathematician with little background in logic. The continuum hypothesis is independent of ZFC. Although `$V = L$' is a 'logical axiom', $\Diamond$ is a 'mathematical axiom'.

3. Let $(G, R)$ be a graph. A subset $Y$ of $G$ is homogeneous if either no pair of elements from $Y$ are in the relation $R$ or all pairs are.

Prove the infinite Ramsey theorem: if $G$ is infinite then $G$ has an infinite homogeneous subset $Y$.

Deduce the finite Ramsey theorem: For every integer $m$ there is an integer $n$ such that if $(G, R)$ is a graph of cardinality at least $n$, it has a homogeneous subset of size $m$. (It may be helpful to expand the language to do this argument.)

4. Prove that every successor cardinal is regular.

5. Let $x < y$ be a well founded partial ordering of some set with the property $(\forall u)[u < x \iff u < y] \rightarrow x = y$. Show $x < y$ is a linear order.

6. For infinite cardinals $\lambda$, prove $\lambda$ times $\lambda$ equals $\lambda$. Does your proof rely on the axiom of choice?