Prelim Exam:

\textit{Answer any six}

Problems from Game Theory

I. Two football teams are after the following 4 players, named A,B,C,D. Suppose the two teams have the following true preferences: Team I: $A > B > C > D$
Team II: $B > C > D > A$
Players are chosen as follows: From available players Team I can choose any one they want. The next turn is for Team II. They can choose any one from the available list. The process terminates once two players for each side are chosen. If the two teams knew the above preferences of each other, show that choosing the best available candidate is not a Nash equilibrium strategy for Team I.

II. From cards numbered 1,2,and 3 players I and II each gets one random card. Seeing his card, player I can say Hi or Lo. After seeing his card and listening to Player I's call player II says Lo or Hi. If the two calls are same the game is over and the one with higher numbered card gets $\$1$, in case both bid Lo, and $\$2$ in case both bid Hi. However, if I bids Hi and II bids Lo then II pays I $\$1$. In case I bids Lo and II bids Hi, then I can either forfeit $\$1$ to II or raise the bid to Hi. Then it is treated as though they both bid Hi. Eliminate an action for each player for each card he may get. What are behavior strategies? Find an optimal behavior strategy for each player.

III. A green snake of length $T$ is sleeping on a green linear tree branch. The branch is 1 unit long. A snake charmer shoots at a point along the branch. In case it hits the snake, the charmer gets $\$50$ worth of its skin. Otherwise, the snake escapes permanently and the snake charmer goes empty handed. Show that the value of the game is $50(1 - \frac{1}{[T]})$. Here $[T]$ denotes the integral part of $T$.

IV. Tom and Dick currently a unit distance apart are approaching each other with one pebble in each one's hand. The one who successfully hits the opponent wins $\$1$ from the opponent. In case they are $x$ units apart, the probability for Tom to hit Dick is $P(x)$, if he decides to use the pebble. The accuracy is $Q(x)$ for Dick. Find an optimal pure strategy for the two players assuming that $P(x)$ and $Q(x)$ are strictly decreasing functions with $P(0) = Q(0) = 1$ and $P(1) = Q(1) = 0$.

V. Players I, II, and III each have two pure strategies. If $K_t(i,j,k)$ is the payoff to player $t$ when pure strategies $i,j,k$ are chosen by players I,II, and III respectively. Suppose the game has $\{(p, 1-p), (q, 1-q), (r, 1-r)\}$ is a Nash equilibrium point where $0 < p,q,r < 1$. Find a set of equations satisfied by $p,q,r$. 

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VI. Given a pair of real matrices $A, B$ of order $m \times n$ use Brouwer's fixed point theorem to show that there exist probability vectors $x, y$ such that when $x_i > 0$ \((Ay)_i = \max_k(Ay)_k\) and when $y_j > 0$ \((B^Tx)_j = \max_s(B^Tx)_s\).

VII. Let $\mathcal{E}$ be the Nash equilibrium set of a bimatrix game $(A, B)$ We call the game Nash determined if for any pair of Nash equilibria $(x, y), (x', y')$ in mixed strategies, $(x, y'), (x', y)$ are also Nash equilibria for the game. Prove that in this case the Nash equilibrium set is convex. Is the converse true?