

For every question, write out your computations in the exam booklet.

1. Find the derivative of the following functions, do simplify.

(a)  $\ln(x^2 + x + 1)$ ,      (b)  $\cos(\sqrt{x})$ ,      (c)  $\arctan(x)$ .

2. Find the derivatives  $f'(x)$  and  $f''(x)$  for the function  $f(x) = e^{-x} \sin(x)$ .

3. Use implicit differentiation to find the slope of the line tangent to the curve

$$xy^2 + 2x^2 - y = 0$$

at the point  $(-1, 1)$ .

4. Let  $f(x) = x^4 - 6x^2 + 4$ .

- (a) Find the critical points and the inflections points of  $f$ .  
(b) On what interval is  $f$  concave down?  
(c) Find the minimum value of  $f$ ?

5. Find the limit:  $\lim_{x \rightarrow 1} \frac{\ln(x)}{x^3 - 1}$ .

6. A family of rectangles in the  $xy$ -plane has one side on the  $x$ -axis, the lower left corner at the origin  $(0, 0)$ , and the upper right corner at a point  $(x, y)$  on the straight line

$$3x + 4y = 5.$$

- (a) Find the area of such a rectangle as a function of  $x$  alone.  
(b) Find the dimensions,  $x$  and  $y$ , of the particular rectangle with the largest area.

