

NAME: \_\_\_\_\_

**You must hand this sheet in with your exam in order to receive a grade.**

**Do not use your calculator.**

1. (15 points) Let  $F(x, y, z) = 4x^2 - y^2 + 3z^2$ . Find the equation of the plane tangent to the level surface  $F(x, y, z) = 7$  at the point  $(1, -3, 2)$ .

2. (20 points) Let  $f(x, y, z) = x^2 - xz + xyz$ .

(a) Find the rate of change of  $f$  at the point  $(1, 1, 1)$  in the direction of the unit vector  $\mathbf{v} = \left(\frac{1}{\sqrt{6}}\right)\langle 2, -1, 1 \rangle$ .

(b) Find the direction in which  $f$  increases most rapidly at the point  $(1, 1, 1)$ , and find the maximum rate of change of  $f$  at that point.

(c) Suppose that the function  $f$  gives the temperature at each point in space. A bug is flying around, with position function  $\mathbf{p}(t) = \langle t, t^2, t^3 \rangle$ , carrying a thermometer in his pocket. Use the chain rule to find the rate of change of his temperature *with respect to time*  $t$  at the moment when his position is  $(1, 1, 1)$ .

3. (15 points) Find the critical points of the function  $f(x, y) = x^4 + y^4 + 4xy - 1$  and classify them as maximum, minimum or saddle points.

4. (15 points) Let  $f(x, y, z) = 1 + x^3 + y^2 - z^3$ . Suppose you were using the method of Lagrange multipliers to find the maximum value of the function  $f$  on the ellipsoid  $x^2 + 3y^2 + 2z^2 = 3$ .

(a) Write down the system of 4 algebraic equation in 4 unknowns that you would need to solve. **Do not try to solve these equations.**

(b) State how you would find the maximum value, given the list of solutions to the equations in part (a).

5. (20 points) Change the order of integration to compute the iterated integral

$$\int_0^3 \int_{\sqrt{x/3}}^1 e^{y^3} dy dx.$$

6. (15 points) Find the surface area of the part of the paraboloid  $z = -x^2 - y^2$  that lies above the plane  $z = -20$ .